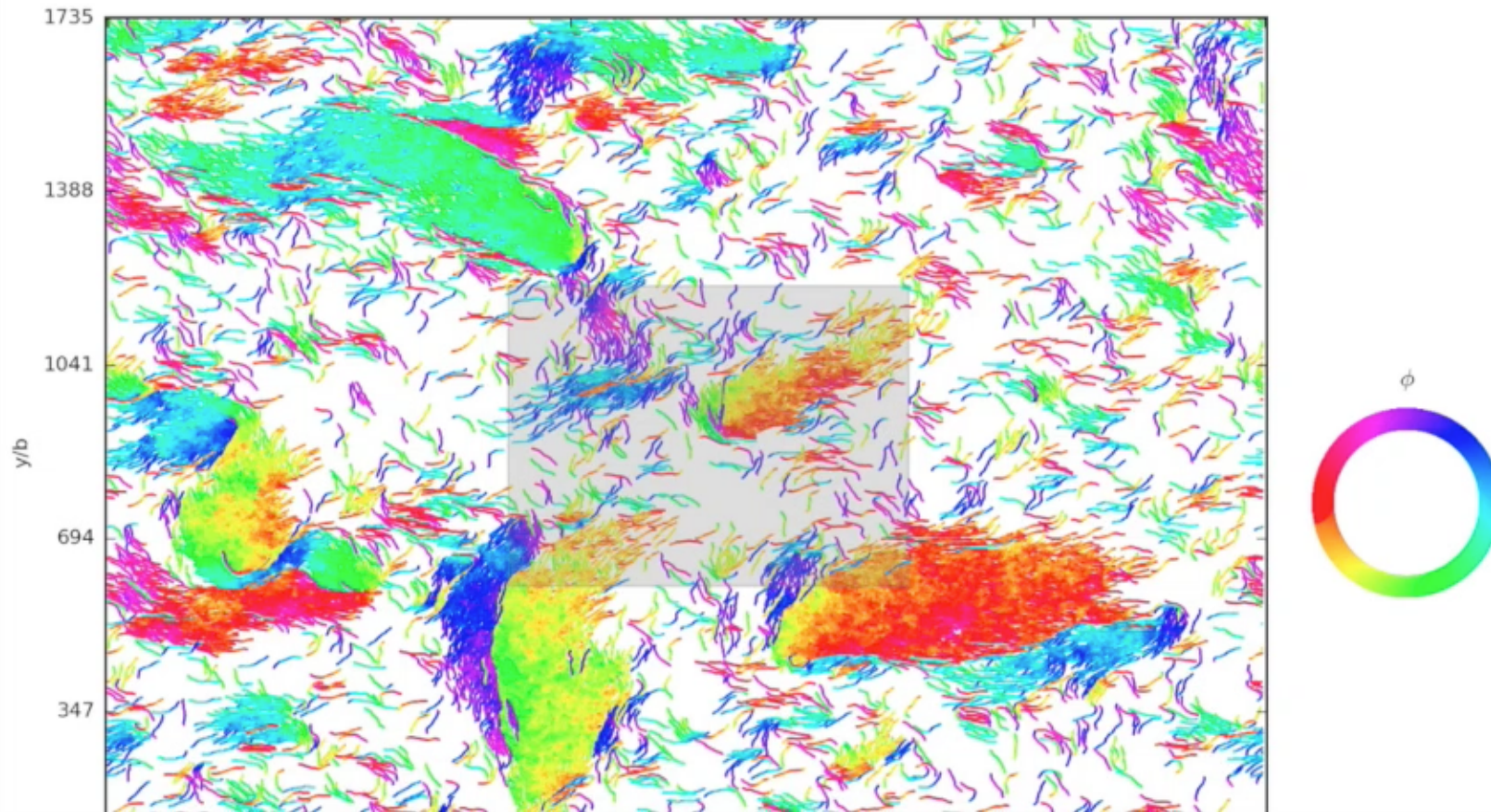




# GROWTH MECHANICS - COEXISTENCE AND EVOLUTION

**JENS ELGETI**

Mitglied der Helmholtz-Gemeinschaft

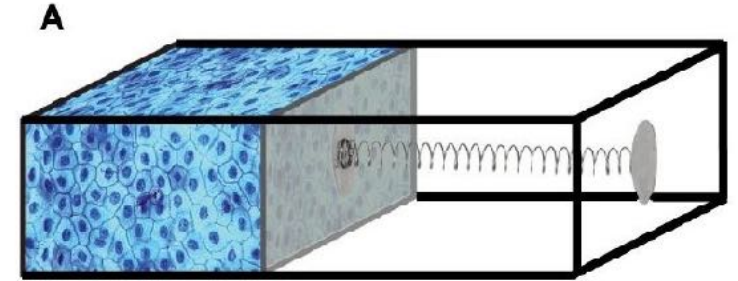
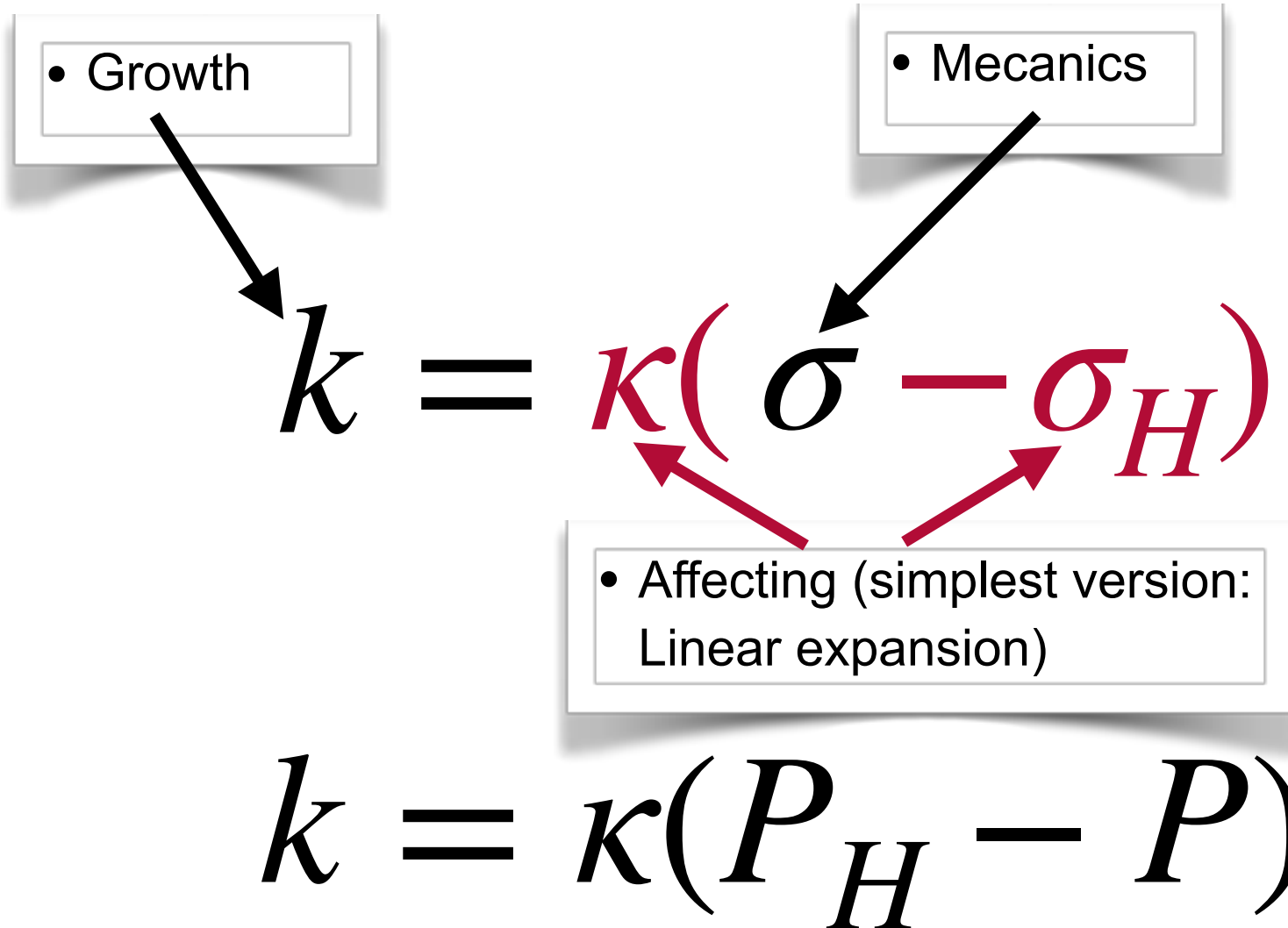
# CANCER IS EVOLUTION WITHIN THE BODY

- Cancer "evolves" with time
- aquirement of new properties
- typically loss of adhesion and enhanced growth (among others)
- Tumor heterogeneity
- Occult Tumors

**• Can Mechanics play a role in Competition & Evolution ???**

go-Jack and Daw Nature Reviews Clinical Oncology 2018

# MECHANICS AFFECTING GROWTH

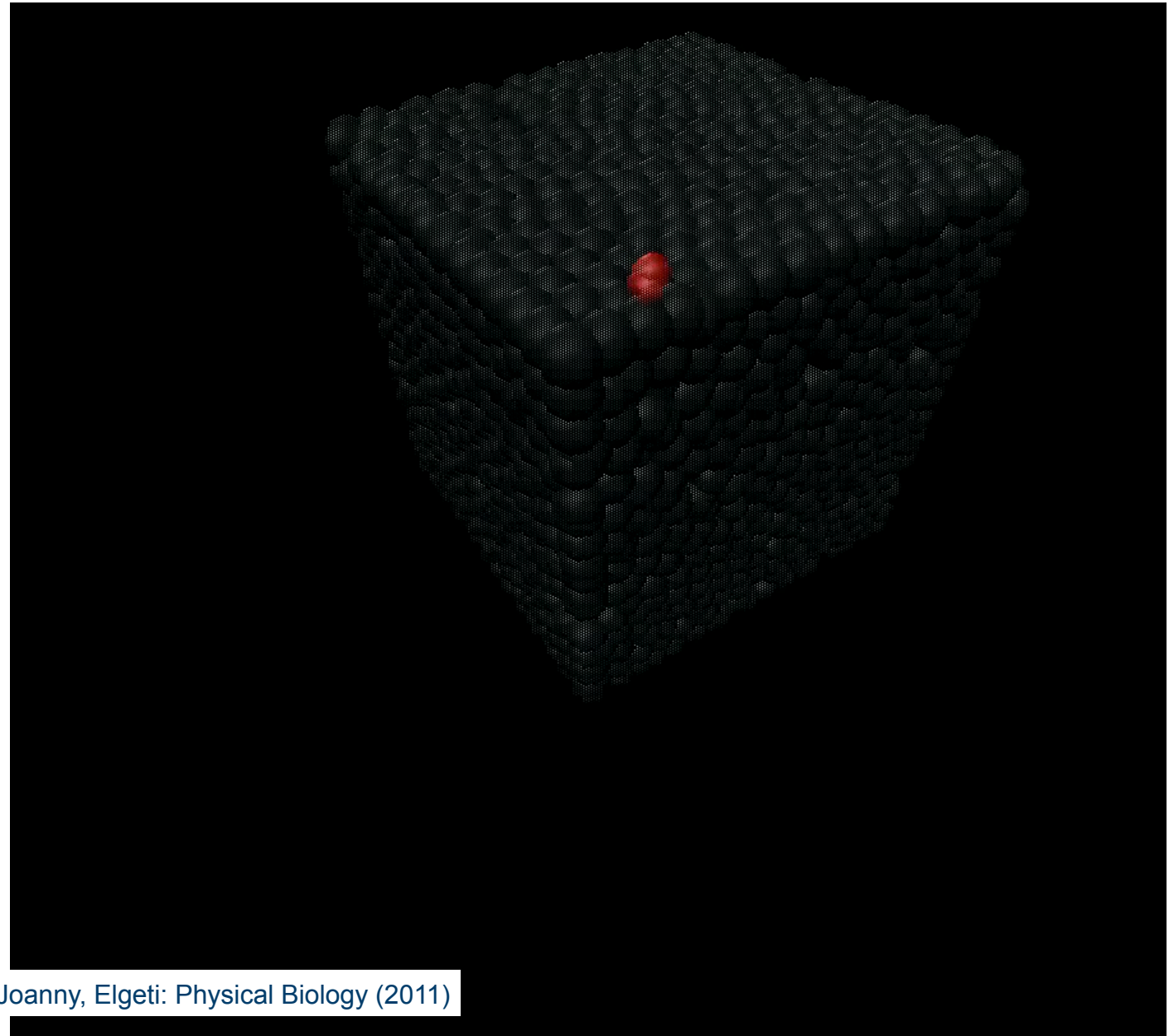


Basan et. al. HFSP 2009

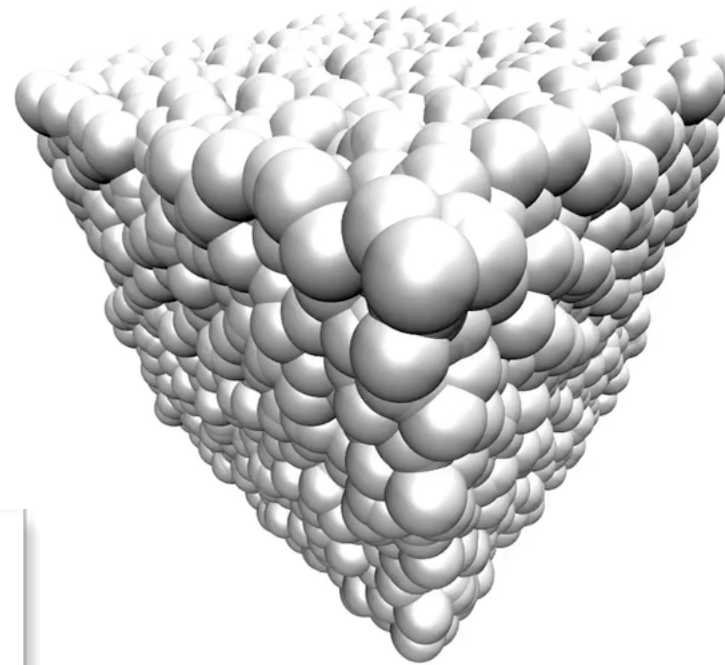
# COMPETITION BY PRESSURE

- Two identical tissues besides homeostatic pressure compete in the same box

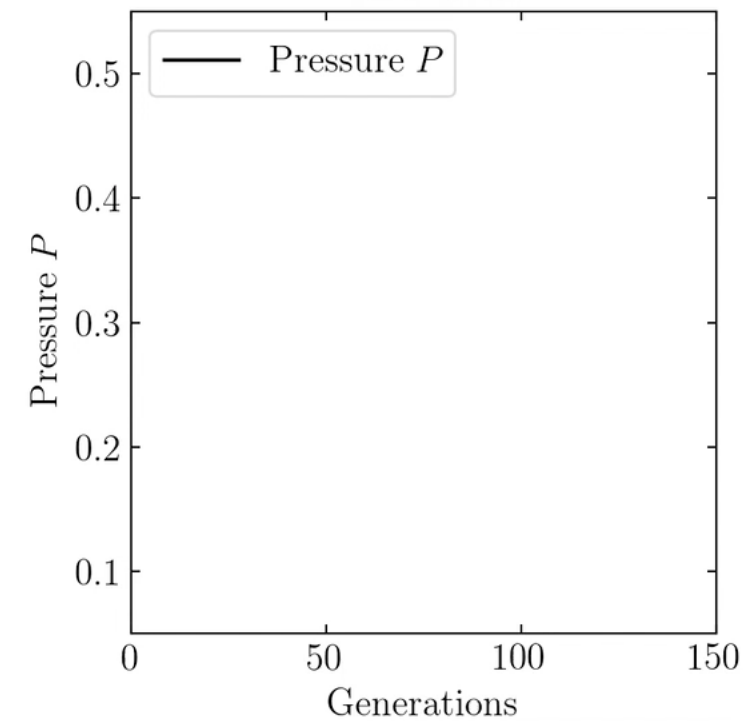
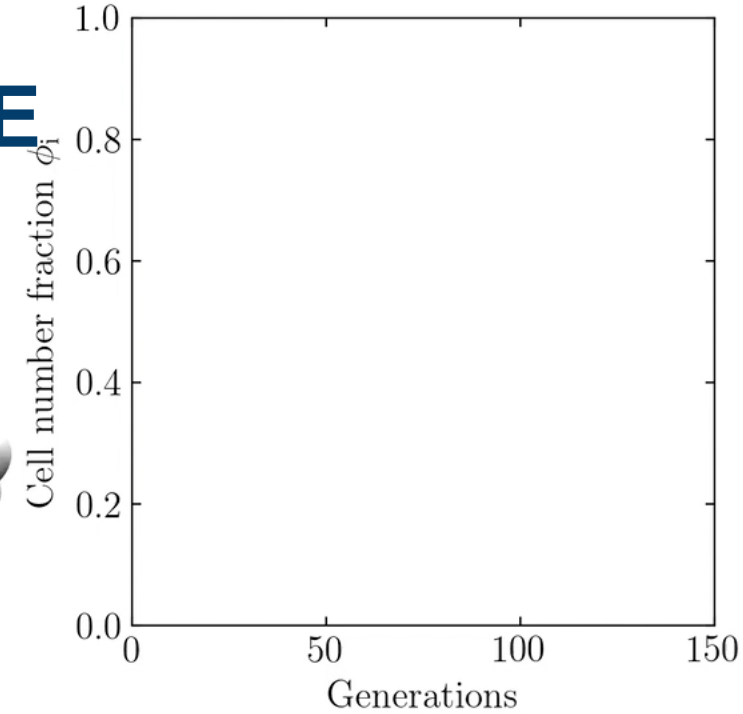
⇒ The stronger wins, even if slower



# EVOLUTION BY HOMEOSTATIC PRESSURE



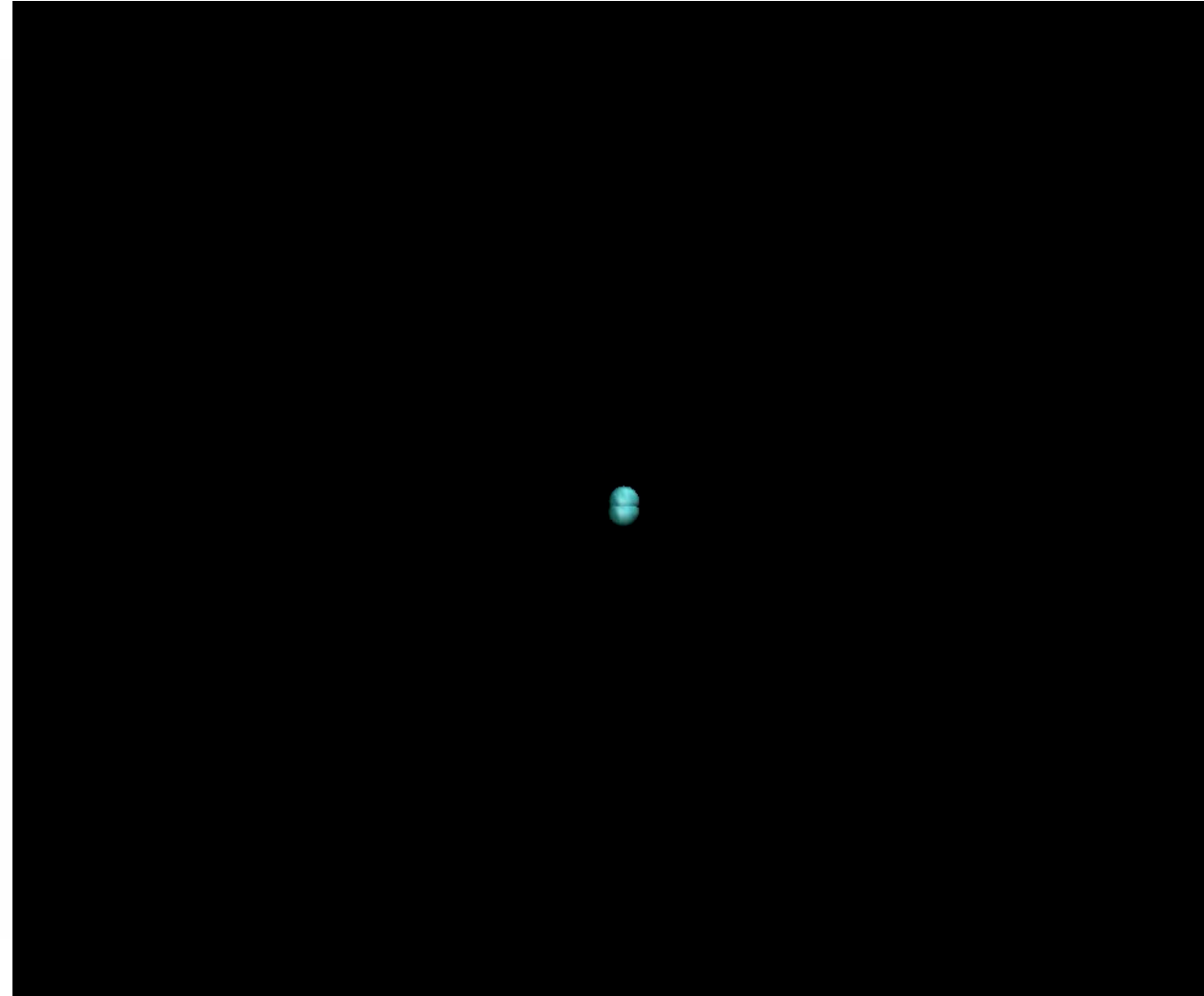
- $k = \kappa(P_H - P)$
- subsequent rounds of takeover



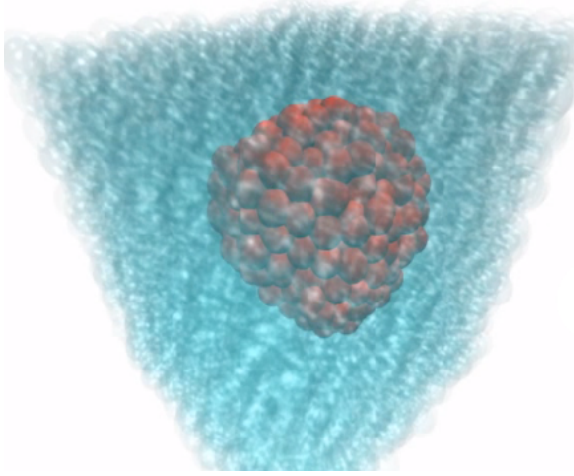
# THE PARTICLE-BASED GROWTH MODEL

## A minimalistic approach

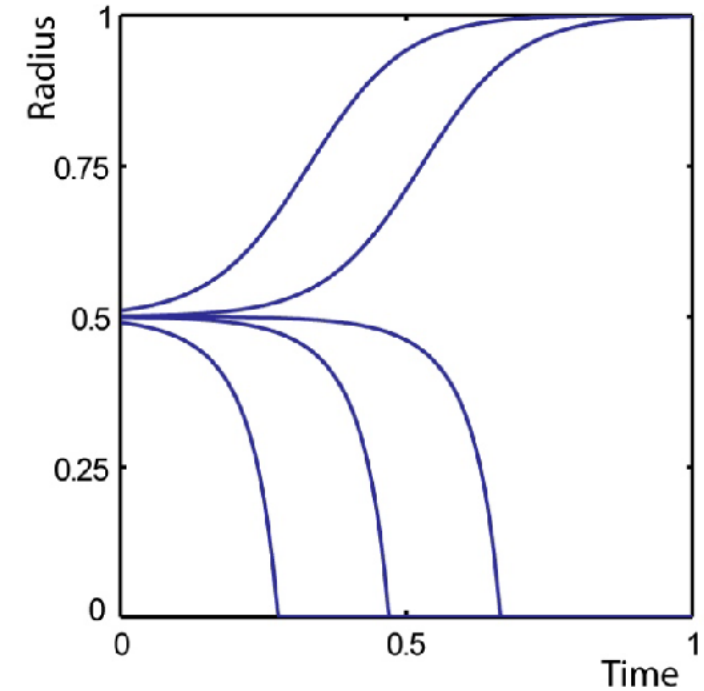
- Cell represented by 2 points
- Active growth force  $G$
- Division at size  
     $\Leftrightarrow$  constant apoptotic rate
- Excluded volume interaction
- Constant attractive force  $f_1$
- DPD thermostat



# METASTATIC INEFFICIENCY & THE ROLE OF ADHESION

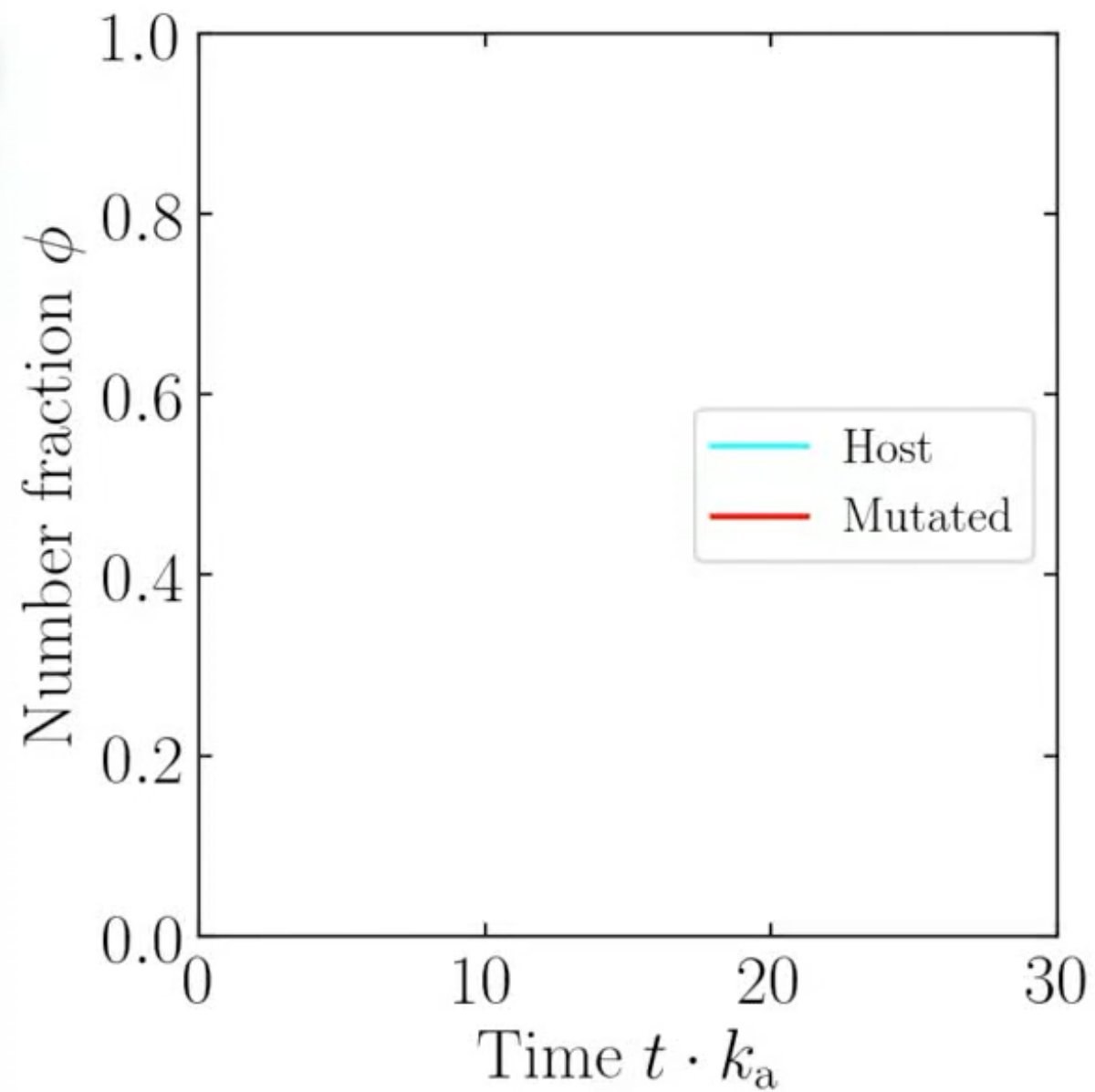
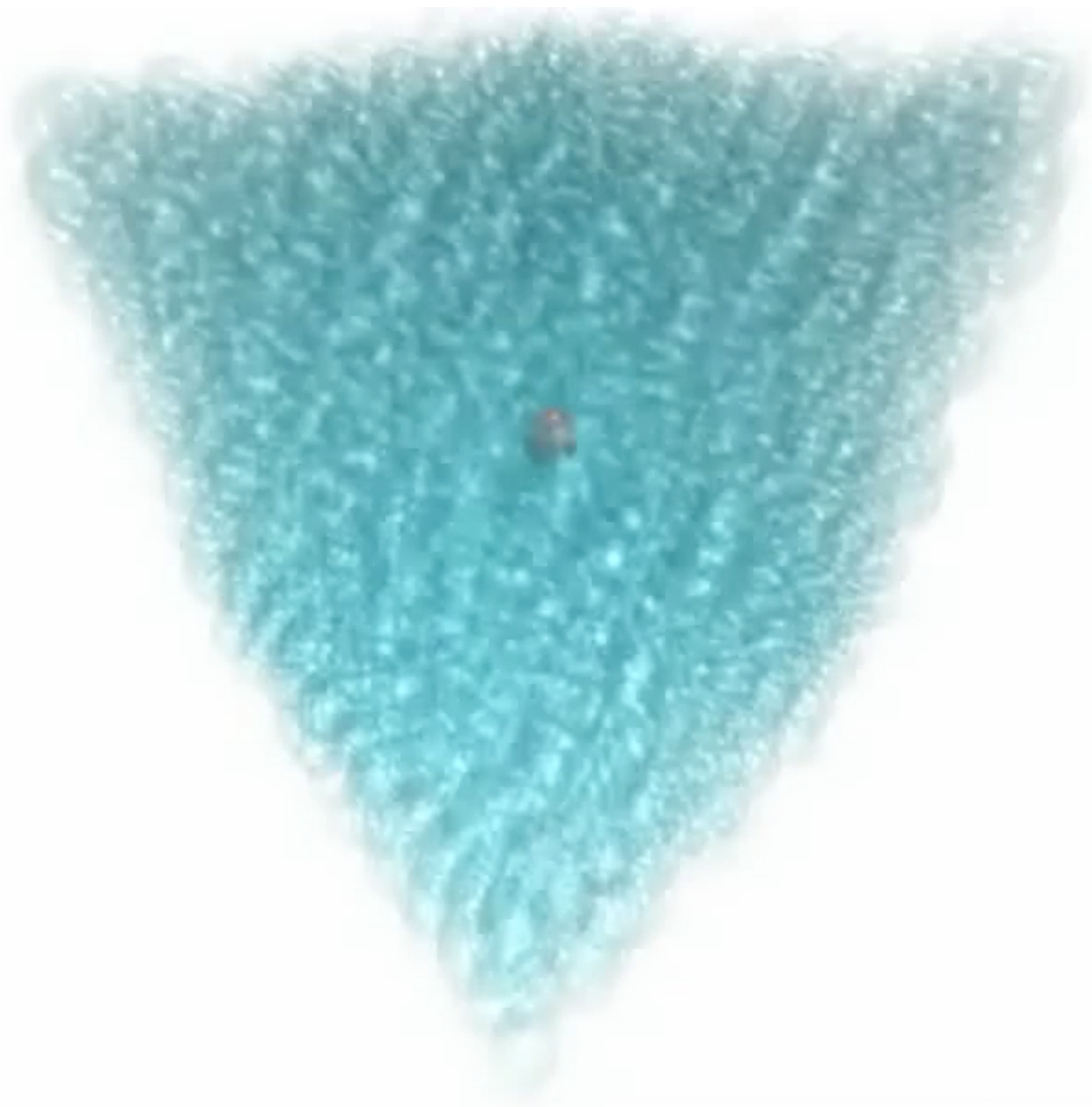


- Surface tension leads to Laplace Pressure  $P = \frac{2\gamma}{R}$
- => Critical tumor size  $R_c = \frac{2\gamma}{\Delta P_H}$



Basan et. al. HFSP 2009

# CRITICAL SIZE?!



# FLAT INTERFACE

## Simplest Case

- Identical Tissues
- Zero Cross adhesion
- Flat Interface

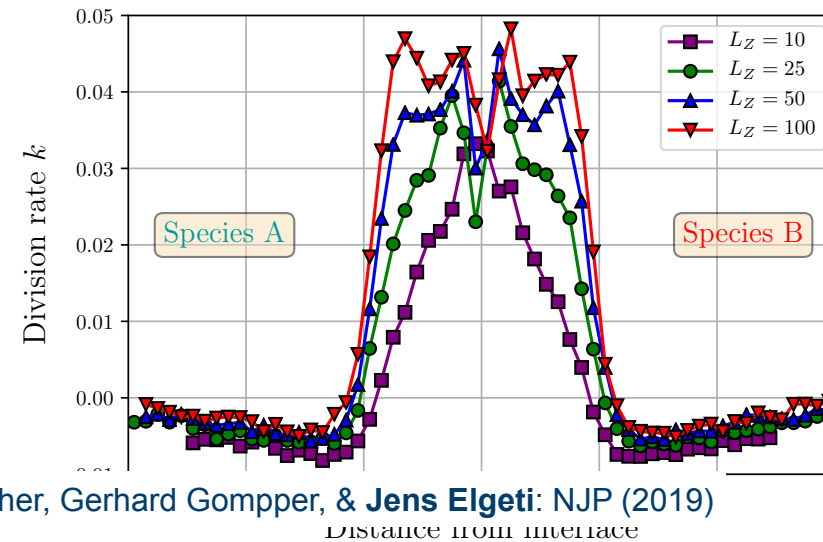
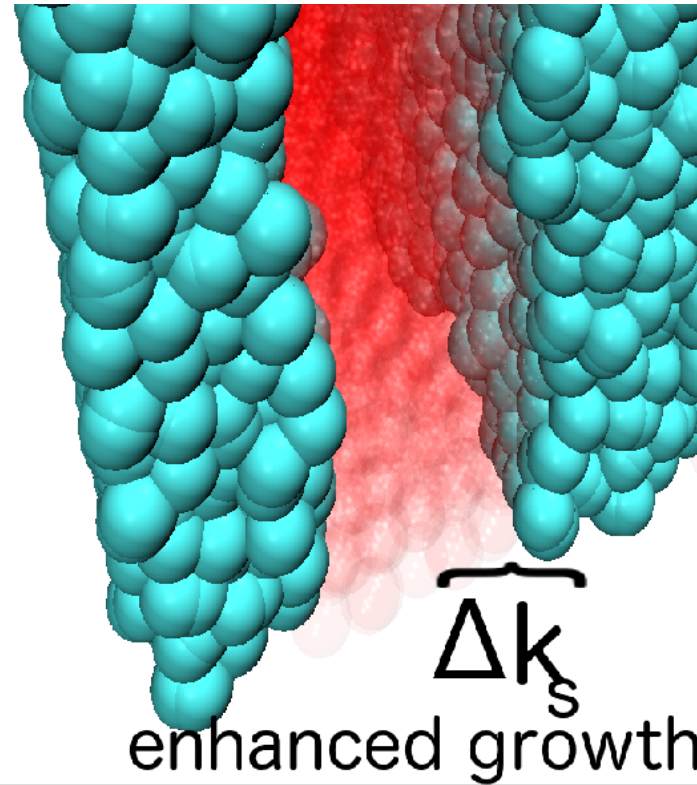
## Explanation

- Zero cross adhesion

⇒ alike to free surface

⇒ interfacial growth

⇒ coexistence



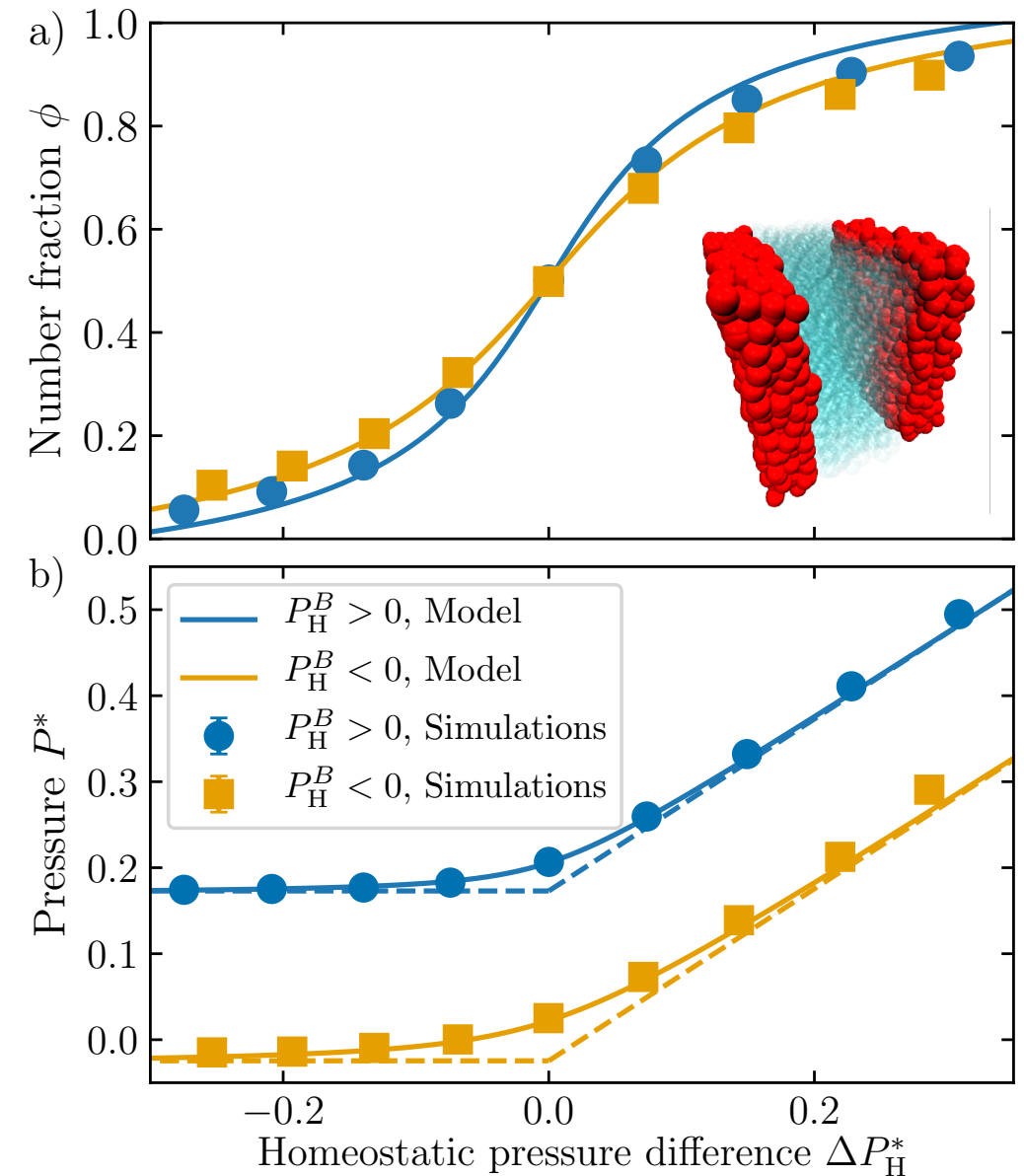
# FLAT INTERFACE

## Theory

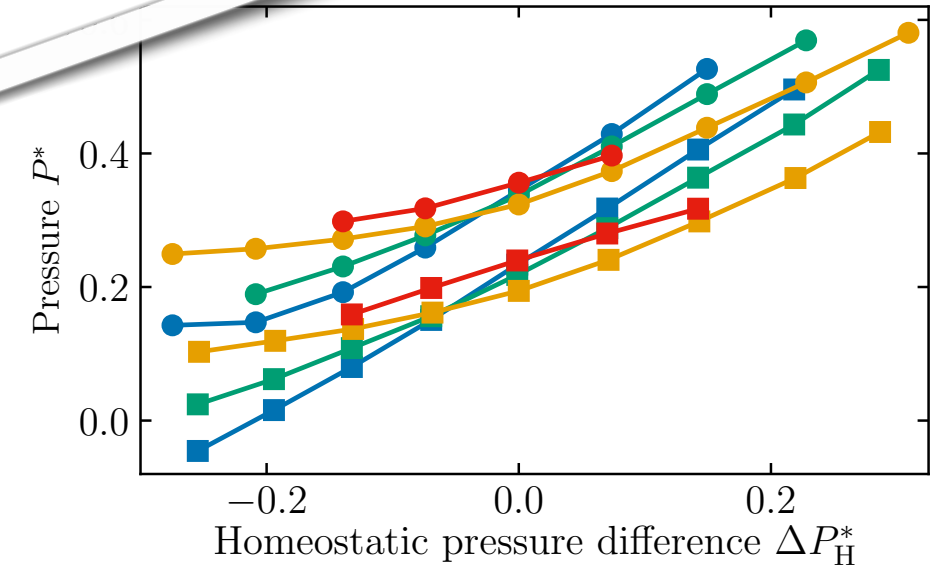
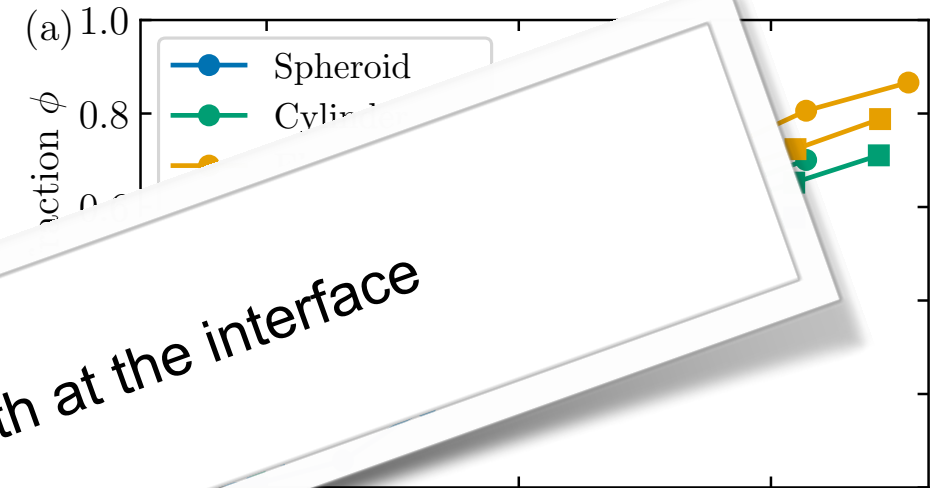
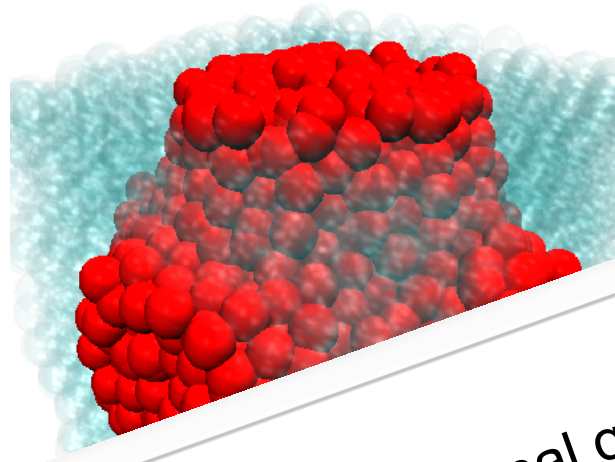
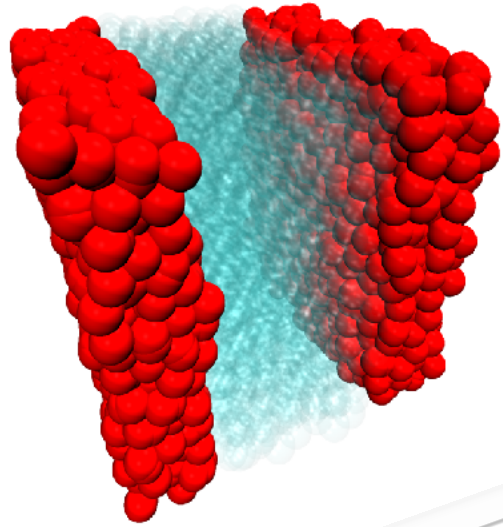
- Homeostatic Pressure:  $k = \kappa(P_H - P)$
- Surface Growth  $\Delta k_s \simeq \Delta k_s^0 + \Delta k_s^1(P_H - P)$
- Number Balance =>

$$P = P_H + \frac{4a\Delta k_s^0}{(4a\Delta k_s^1 + \kappa L_z)}$$

- Balance pressures
- Pressure higher than hom. P of stronger tissue!
- Cell death balanced by excess growth at surface.



# COMPLEX STRUCTURES



Bottom Line:

- Lag of Cross adhesion can lead to additional growth at the interface
- Interfacial growth stabilizes coexistence

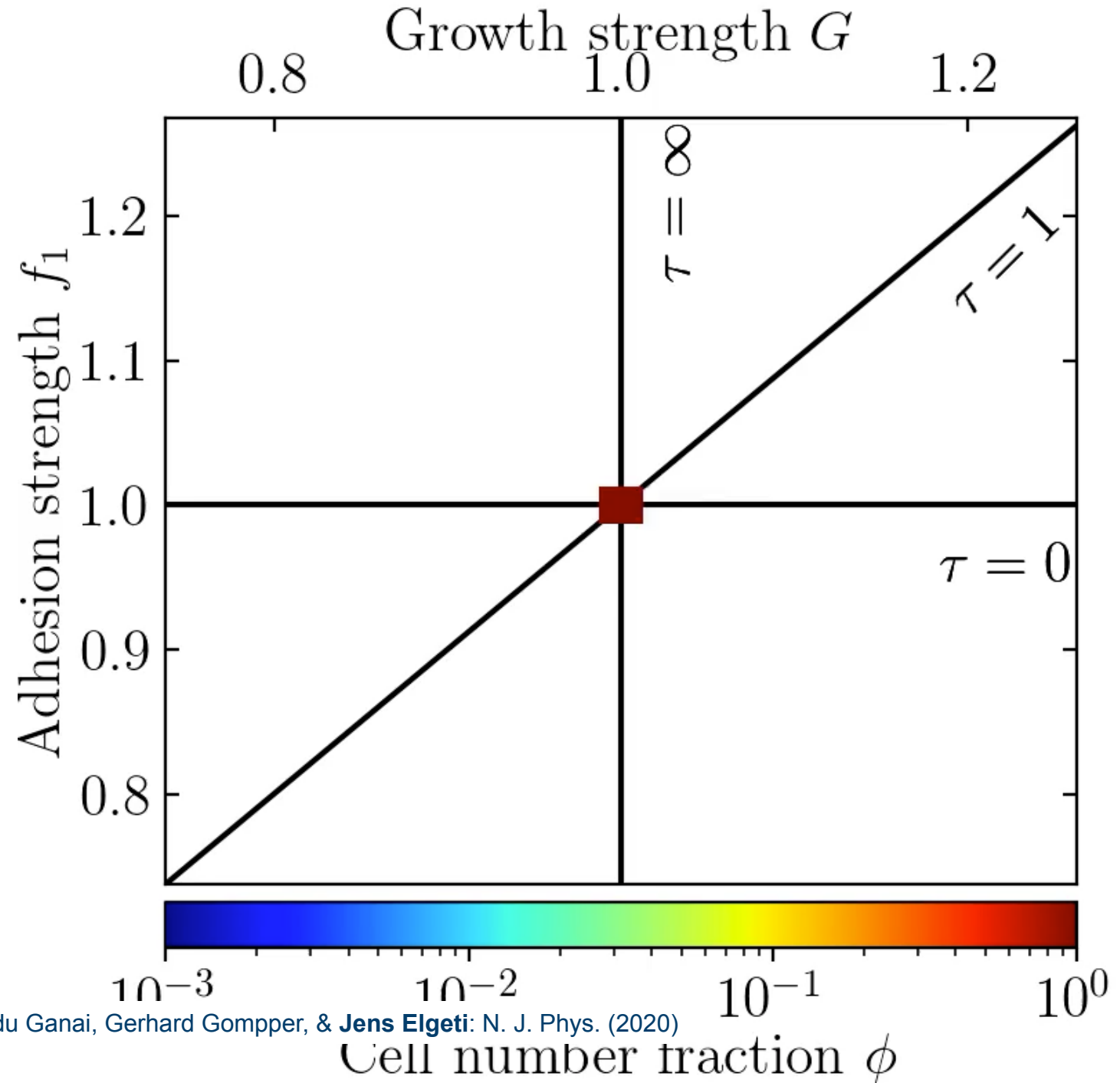
# EVOLUTION

## Evolution of parameters

- Mutation possibility after each division
- Evolves to low adhering, strong growing tissue.

## Coupling of parameters

- Tradeoff  $\tau$  between growth strength and adhesion:  
$$f_1 = \tau G$$
- Motivated by cadherine biology



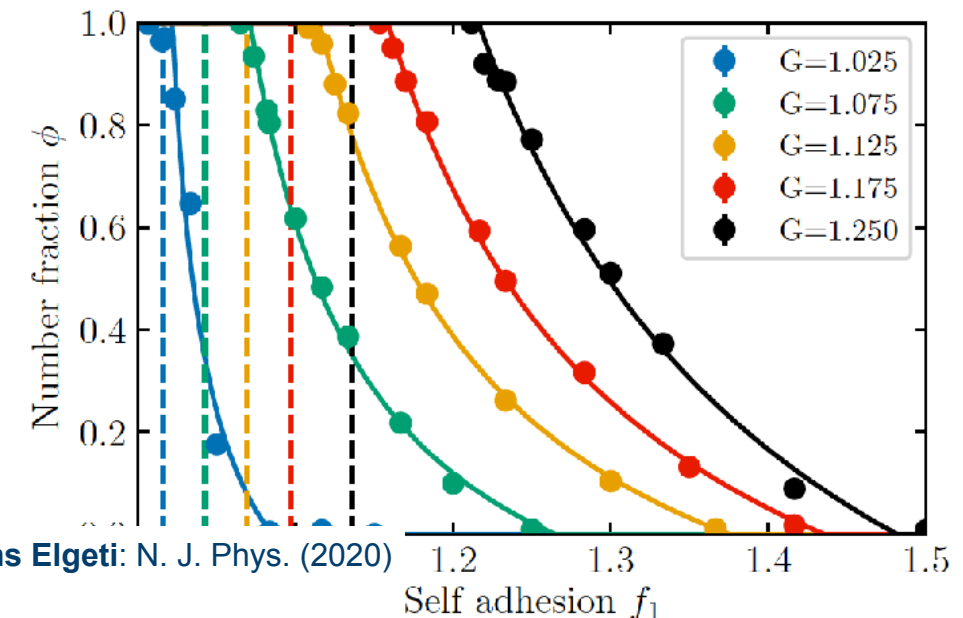
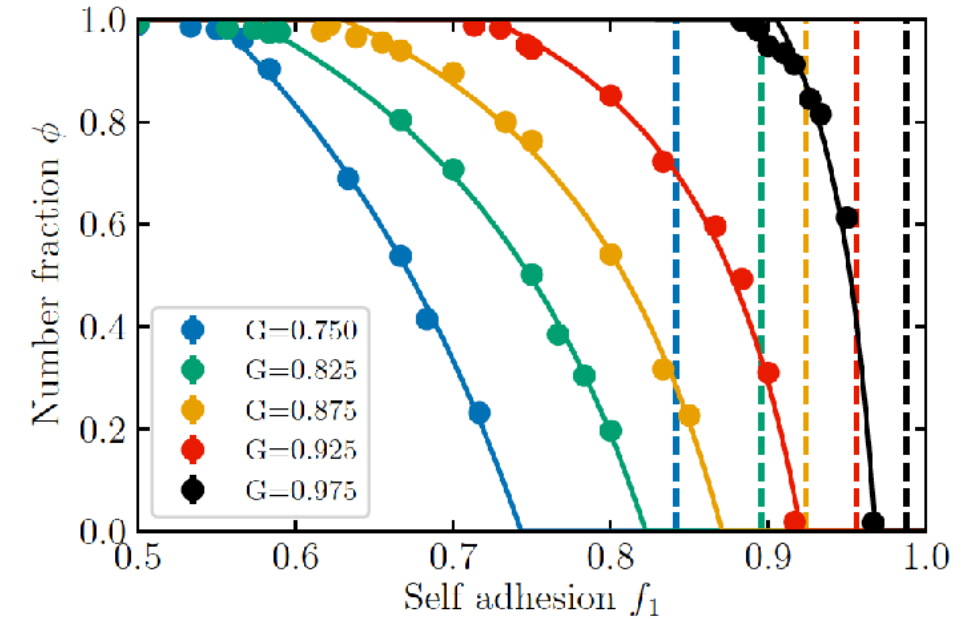
# SMALL DIFFERENCES IN CROSS ADHESION & GENETIC DRIFT

- Assume that cells can mutate, changing cross adhesion and growth strength
- (Imagine mutating Catherines: Adhesion and growth signalling)
- Assume well mixed state, surface growth proportional to  $\phi(1 - \phi)$
- Number balance:

$$\partial_t \phi = \kappa(P_H^A - P)\phi + \Delta k_s^A \phi(1 - \phi) \quad (2)$$

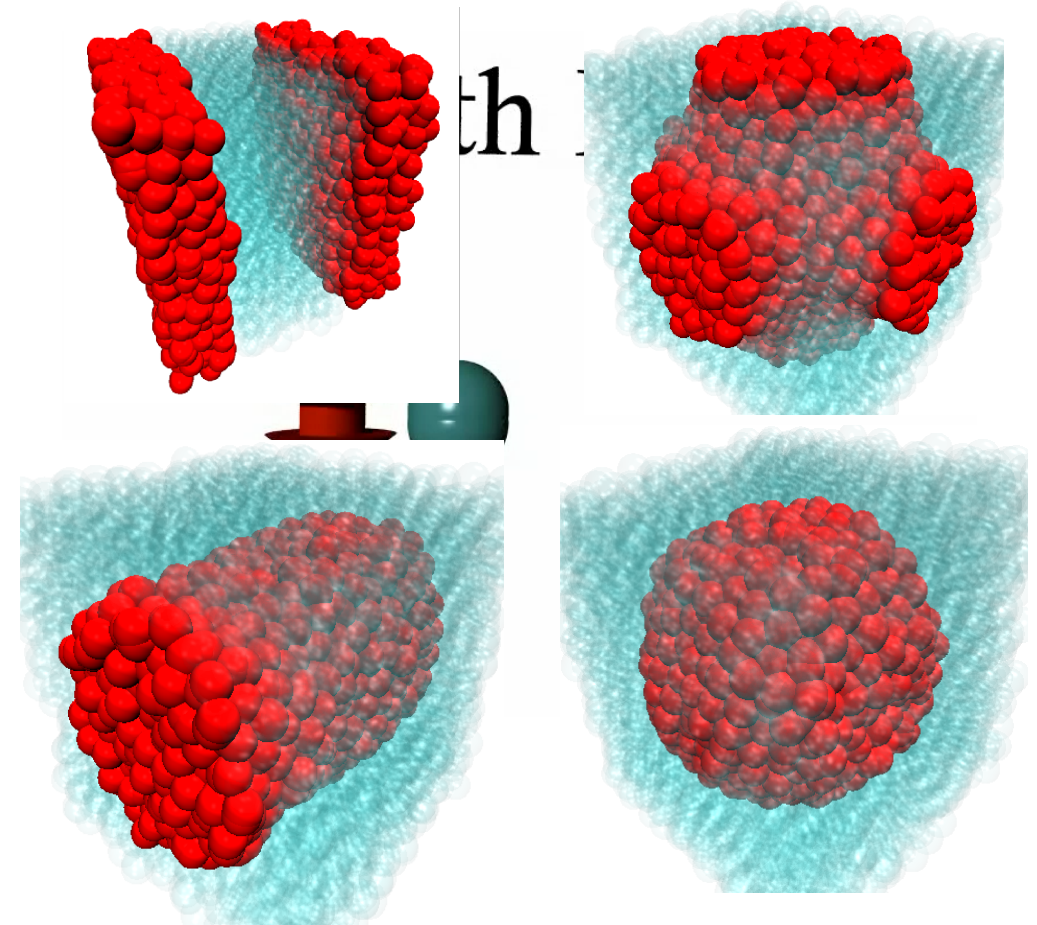
$$\partial_t (1 - \phi) = \kappa(P_H^A + \Delta P_H - P)(1 - \phi) + \Delta k_s^B \phi(1 - \phi),$$

$$\phi_3 = \frac{\kappa \Delta P_H + \Delta k_s^A}{\Delta k_s^A + \Delta k_s^B}$$



# INTERMEDIATE SUMMARY

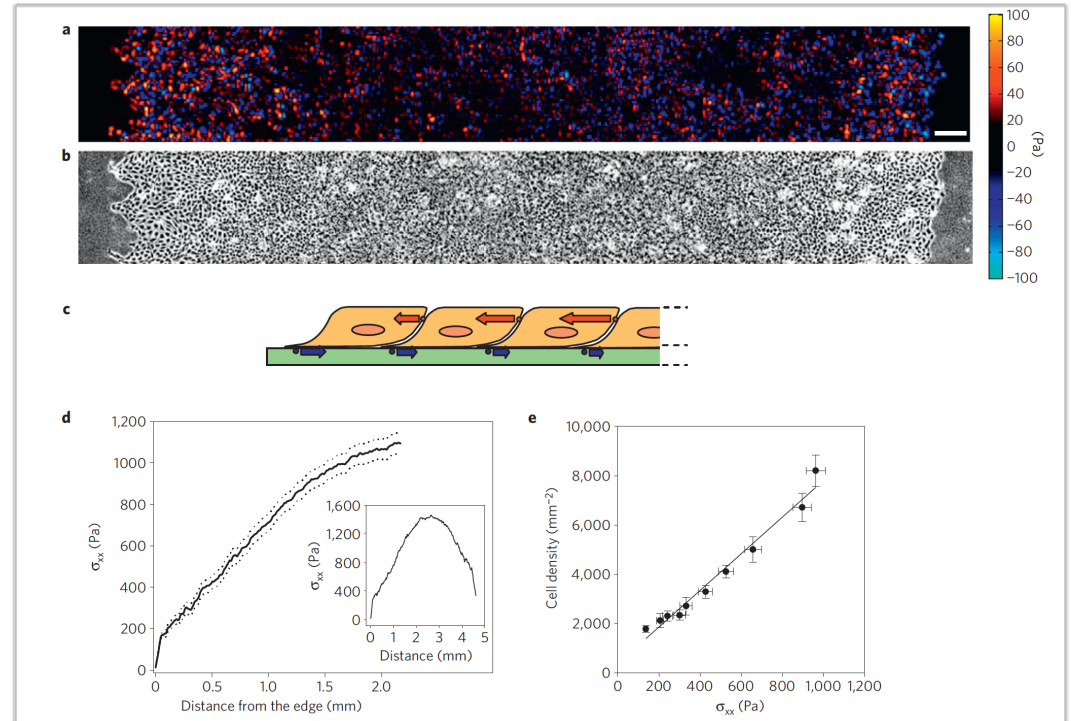
- Mechanics can control tissue growth and competition
- Homeostatic Pressure Key control parameter
- Surface and Interface Effects lead to additional complexity
- Coexistence and divergent evolution possible



# MOTILE (GROWING) TISSUES

- Many motile cells move "fluid like", but remain cohesively in a colony.
- corresponds to "Fluid-vacuum" coexistence

- Furthermore the Colony is under ***tension***



Trepat et al. (2009). Nat. Phys. 5, 426

Puliafito et al. (2012). Proc. Natl. Acad. Sci. USA 109, 739

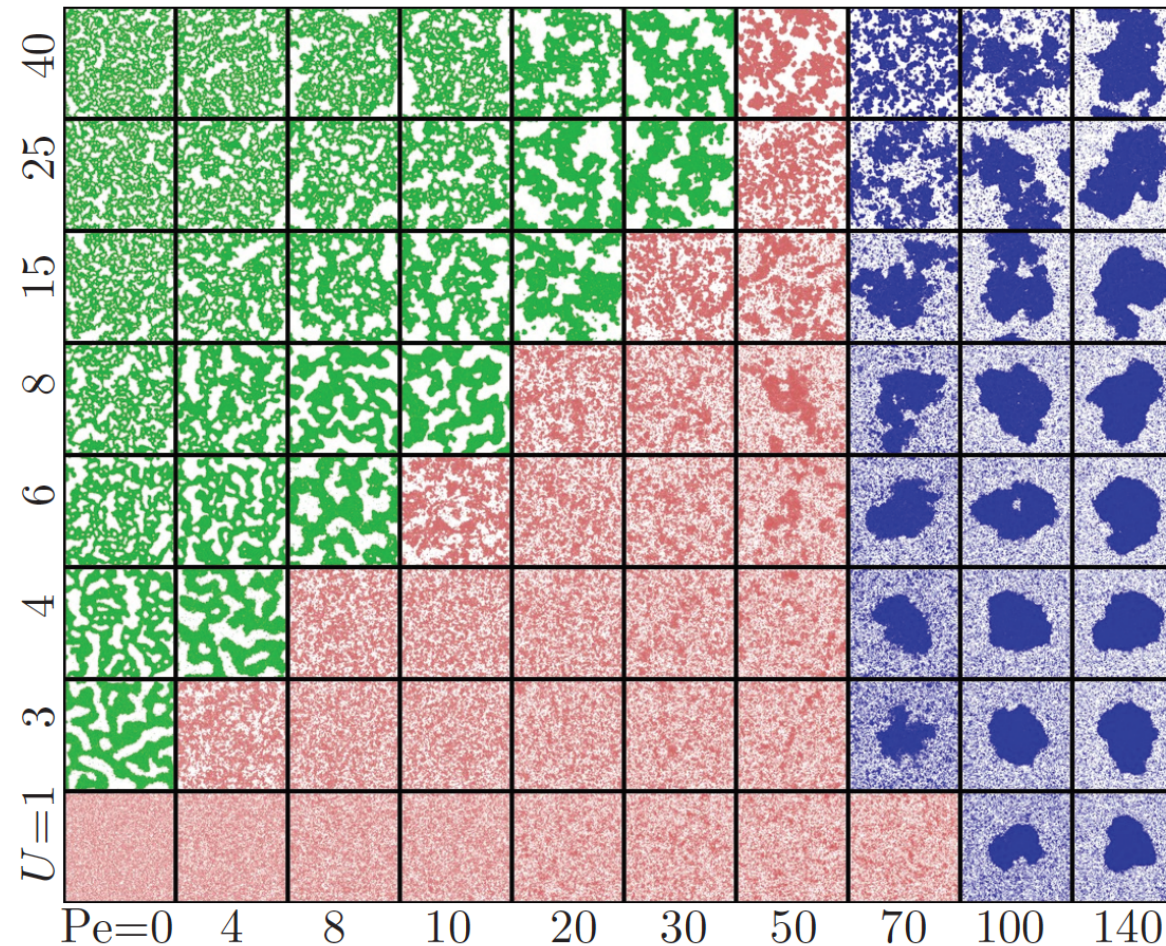
# ACTIVE BRONIAN PARTICLE

## ABP Model

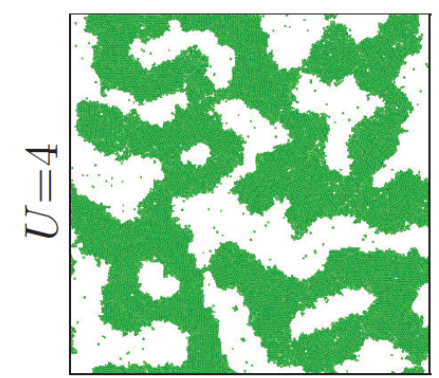
- Brownian Diffusion (incl. rotation)
- Active Propulsion
- LJ Interaction

## Three Situations

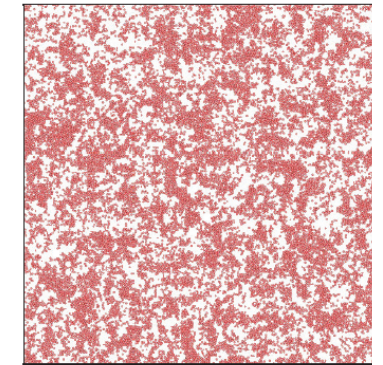
- Solid+Vacuum
- Fluid+Gas
- Gas+MIPS



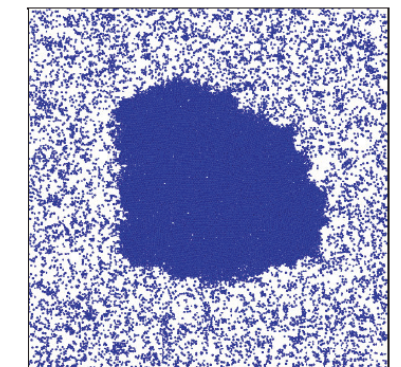
Redner et al. PRE & PRL 2013



Pe=4



Pe=20

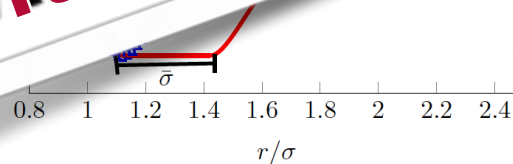


Pe=100

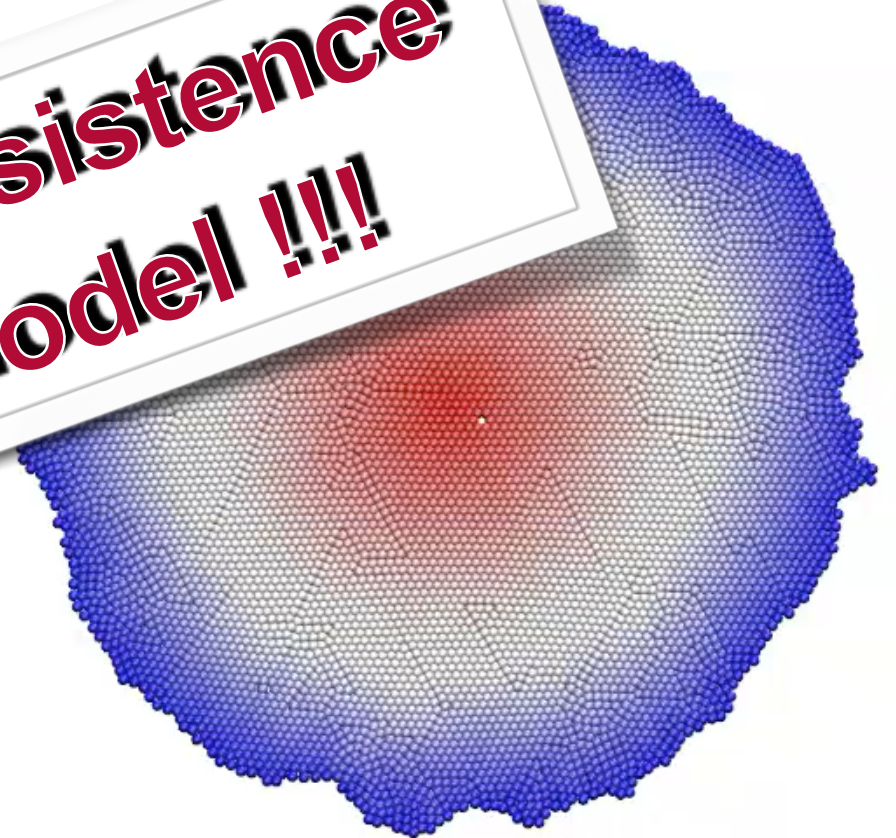
# WIGGLE ROOM

- "Ususual" ABP Model
  - Brownian colloid with orientational and translational diffusion
  - Propulsion along orientation
- Plus **extended** LJ Potential  
pot-width  $\bar{\sigma}$

**Fluid - Vacuum Coexistence  
with minimal ABP Model !!!**



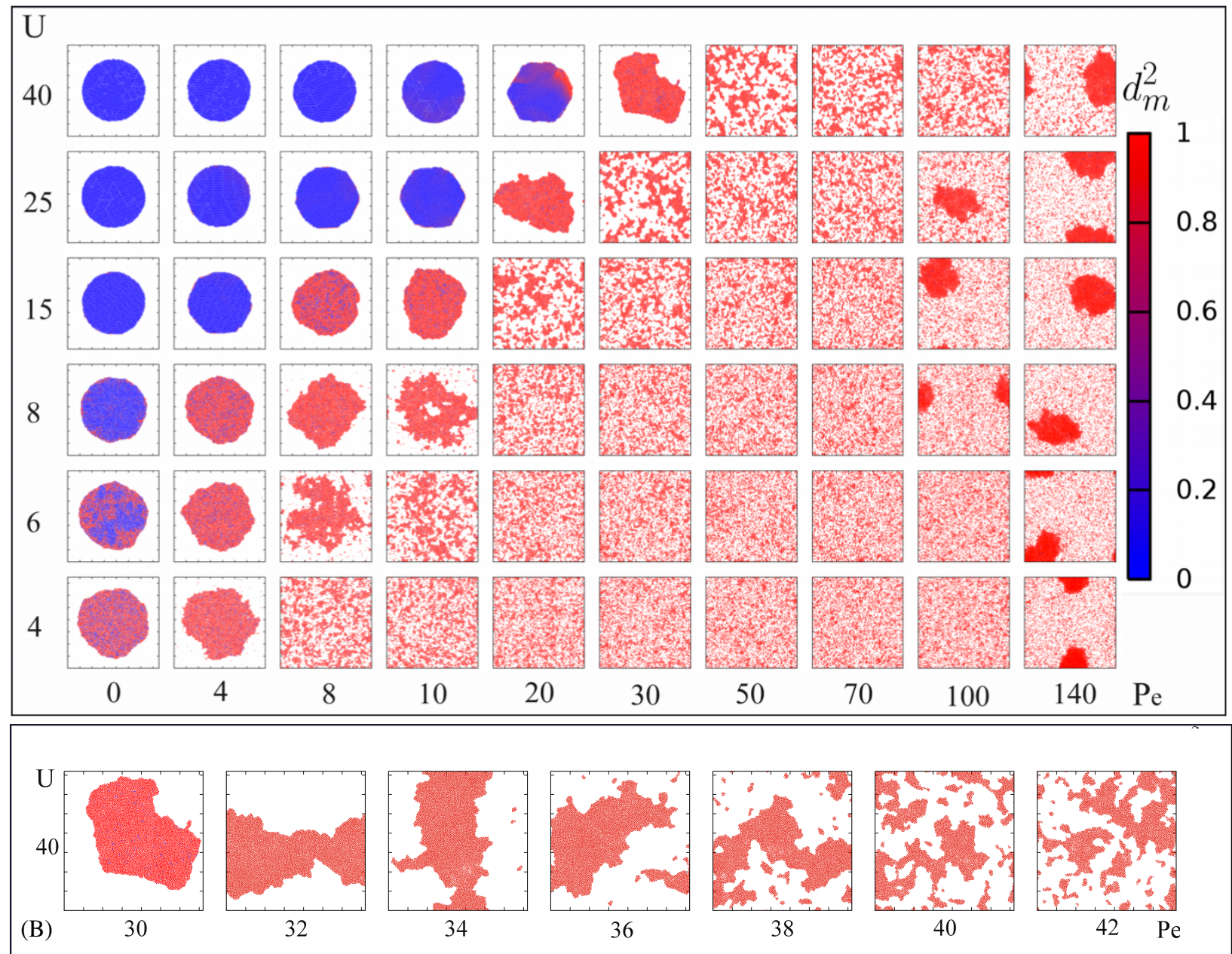
$t=0.0$



# PHASE BEHAVIOR

## Observed "Phases"

- Solid + Vacuum
- Liquid + Vacuum
- rarely small Clusters detach
- many clusters detach
- Many Clusters
- MIPS (compare e.g. Talk of Thomas Speck or Roland Winkler)



# "QUANTIFYING" THE PHASES

## Liquidity:

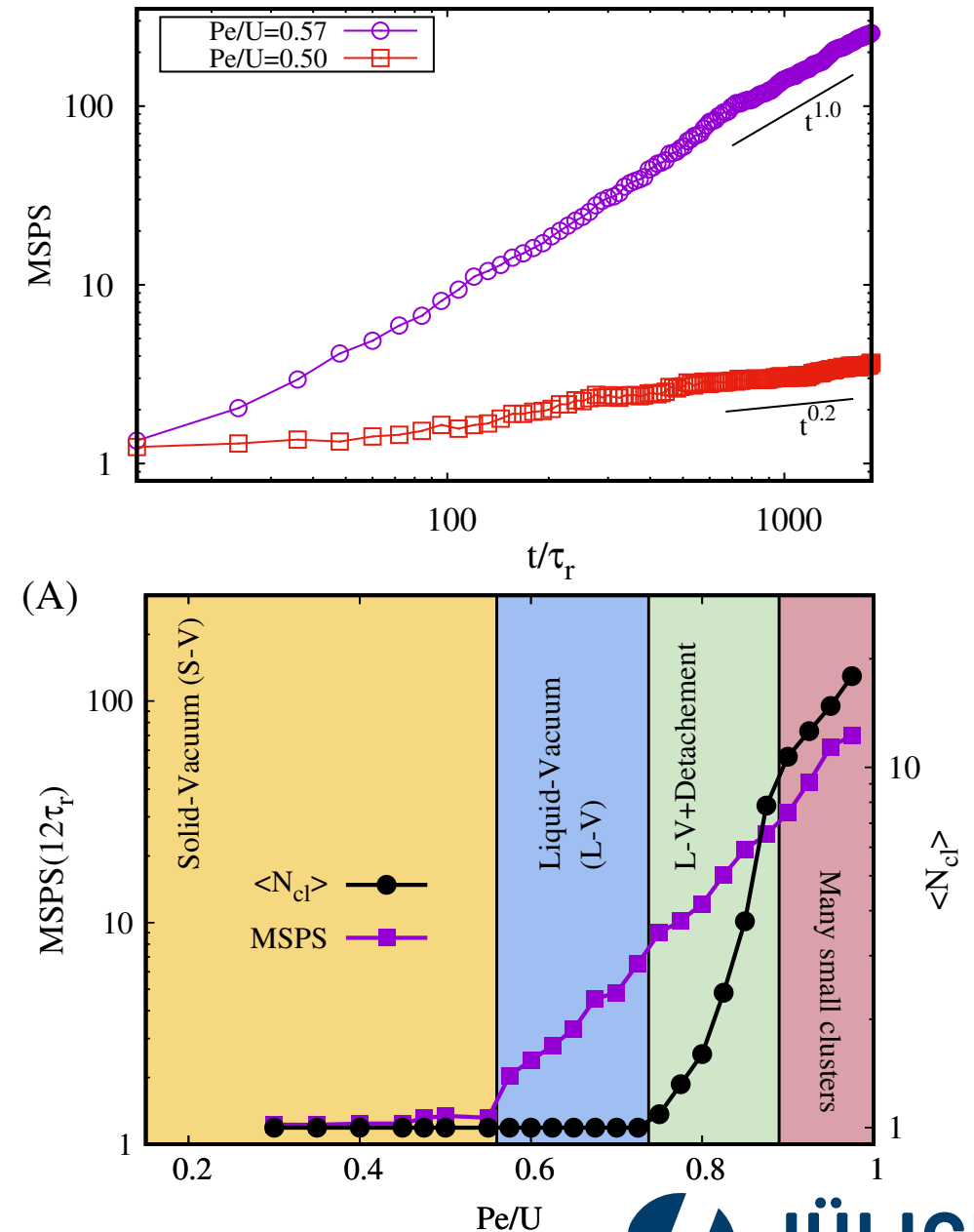
- Liquidity defined by neighbour exchange
- Define Mean squared separation of particle-pairs touching at  $t=0$ :

$$MSPS(t) = \frac{1}{N_p} \sum_{N_p} (\mathbf{r}_m(t_p + t) - \mathbf{r}_n(t_p + t))^2$$

- $MSPS(12\tau_r) > 1.2\sigma^2$  indicates liquidity

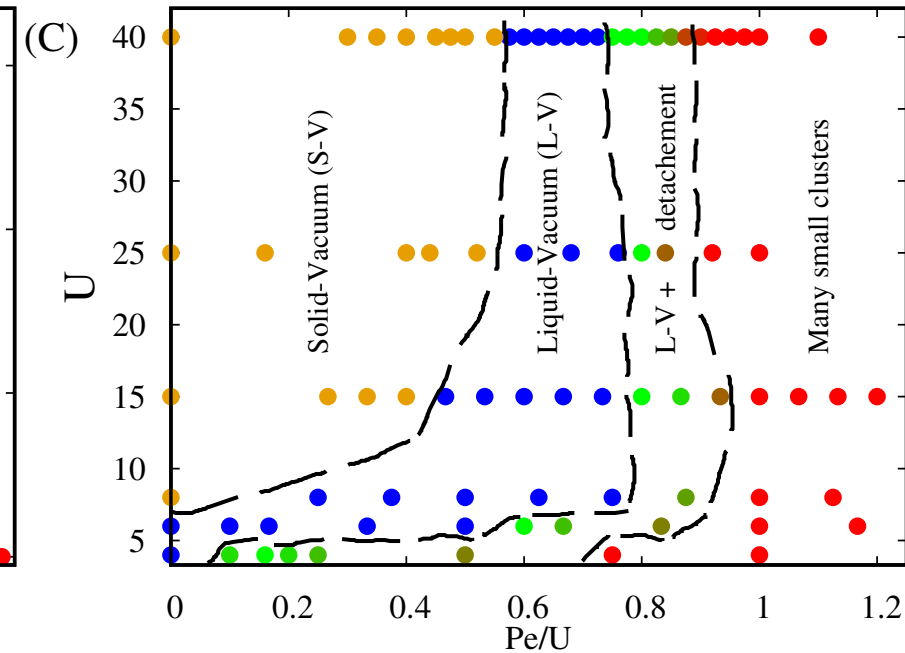
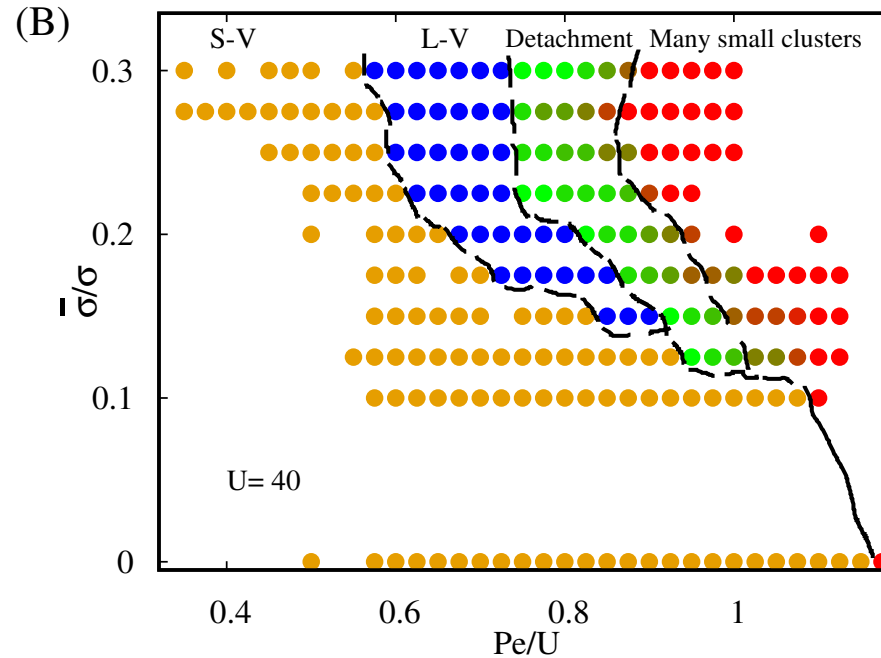
## "Vacuum":

- Number of Clusters = 1 indicates "**vacuum**" coexistence



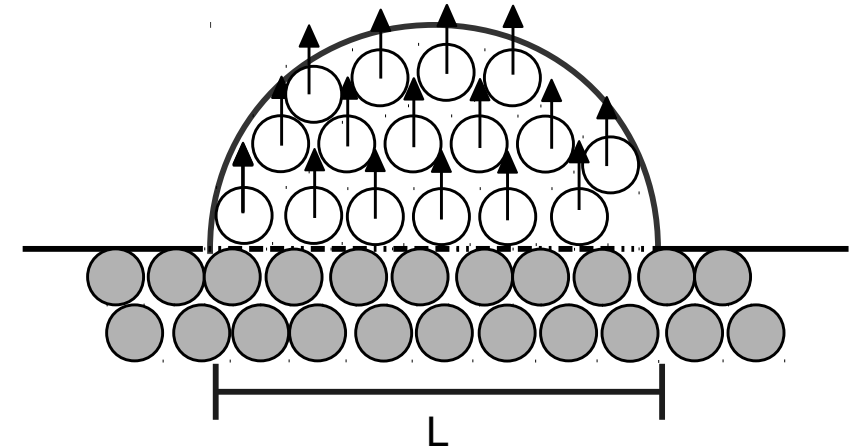
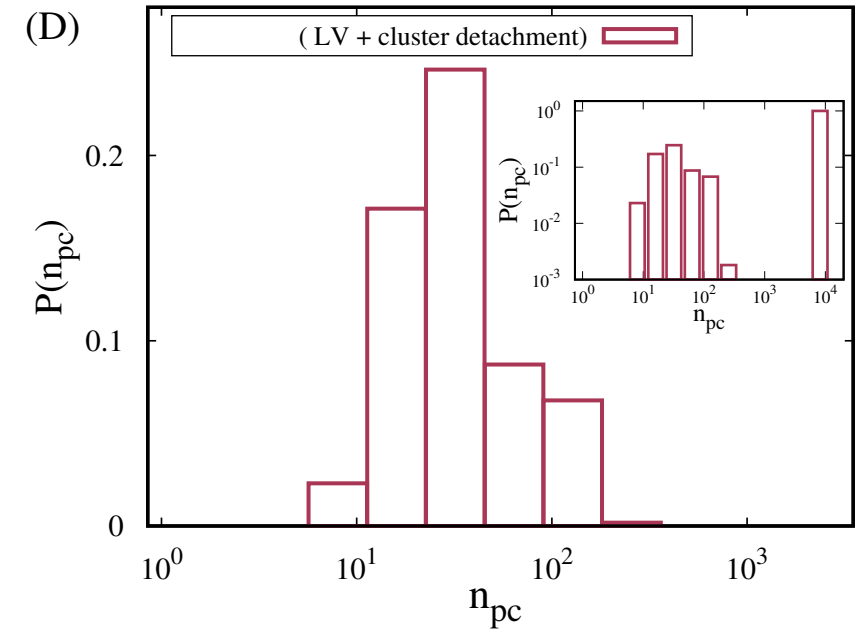
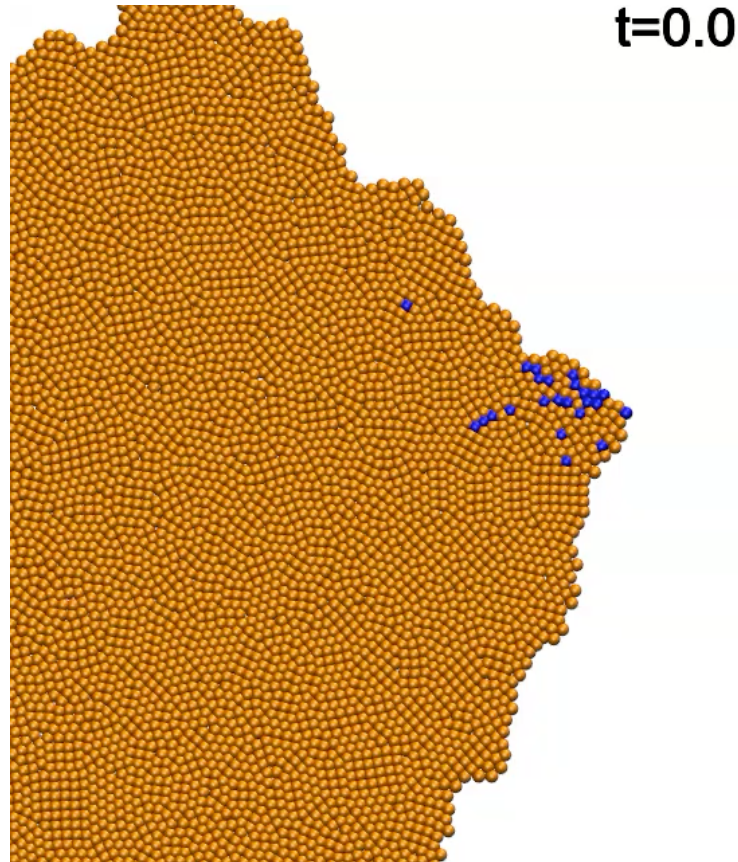
# EXPLORING PHASE SPACE

- Extend of the Liquid-Vacuum phase, close to Activity  $\sim$  Attraction
- Strong adhesion compared to noise required
- Minimal potential width  $\bar{\sigma}$  necessary



# A NOTE ON CLUSTERS

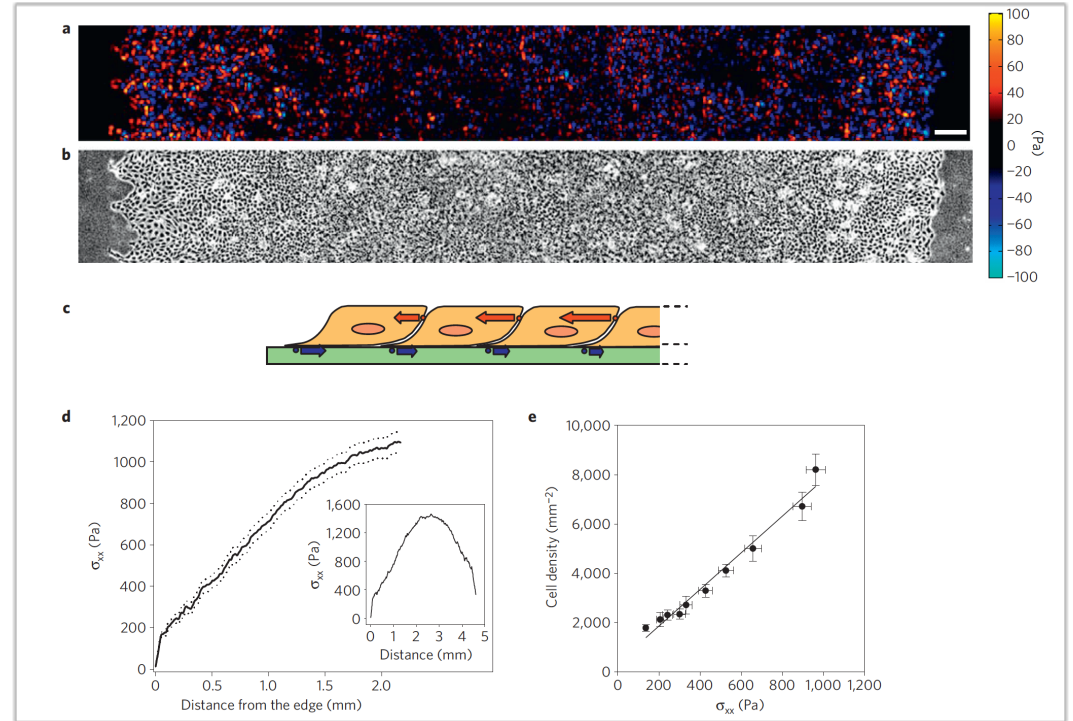
- Cells detach in **clusters**
- No single Cell detaches
- Simple explanation: Single cells to "weak" to detach - aligned can.
- Note that cancer cells also often detach from the tumor in clusters!



# MOTILE TISSUES

- Many motile cells move "fluid like", but remain cohesively in a colony.
- corresponds to "Fluid-vacuum" coexistence

- Furthermore the Colony is under ***tension***



Trepat et al. (2009). Nat. Phys. 5, 426

# STRESS PROFILE

## Polarity

Polarisation is defined as,

$$\langle p \rangle = \frac{1}{N} \sum_{i=1}^N (\hat{\mathbf{n}}_i \cdot \hat{\mathbf{r}}'_i) \text{ here, } \mathbf{r}'_i = \mathbf{r}_i - \mathbf{r}_{cm}$$

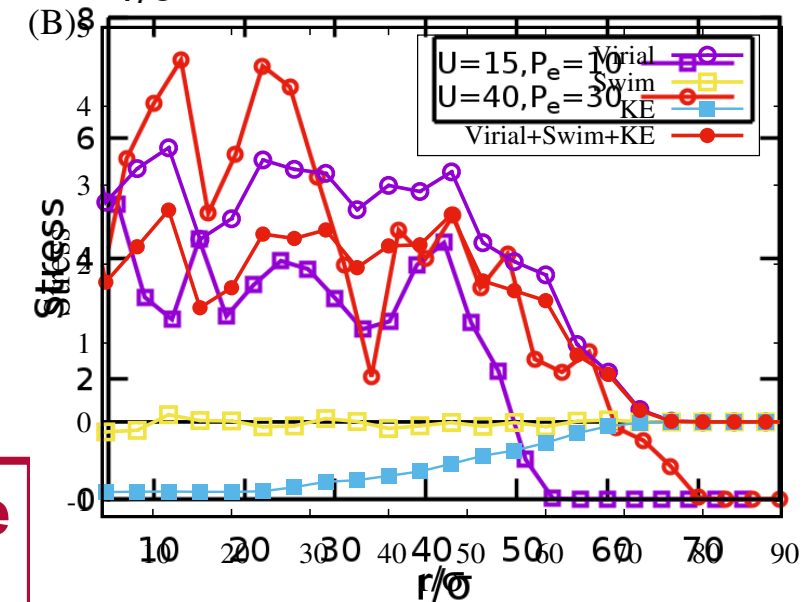
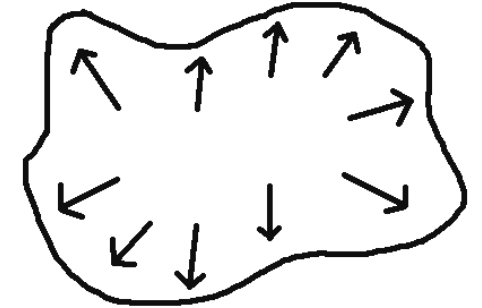
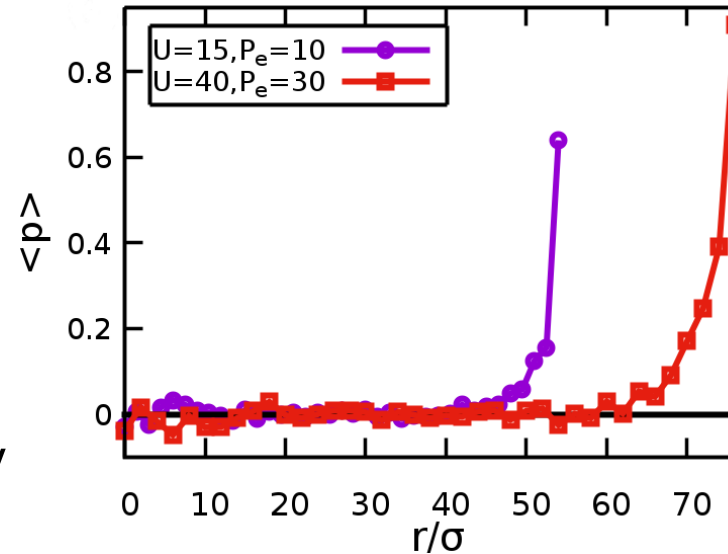
- Due to fluidity, **outward** alignment of mobility forces at the boundary of cell colony

## Stress profile

Stress  $\Omega$

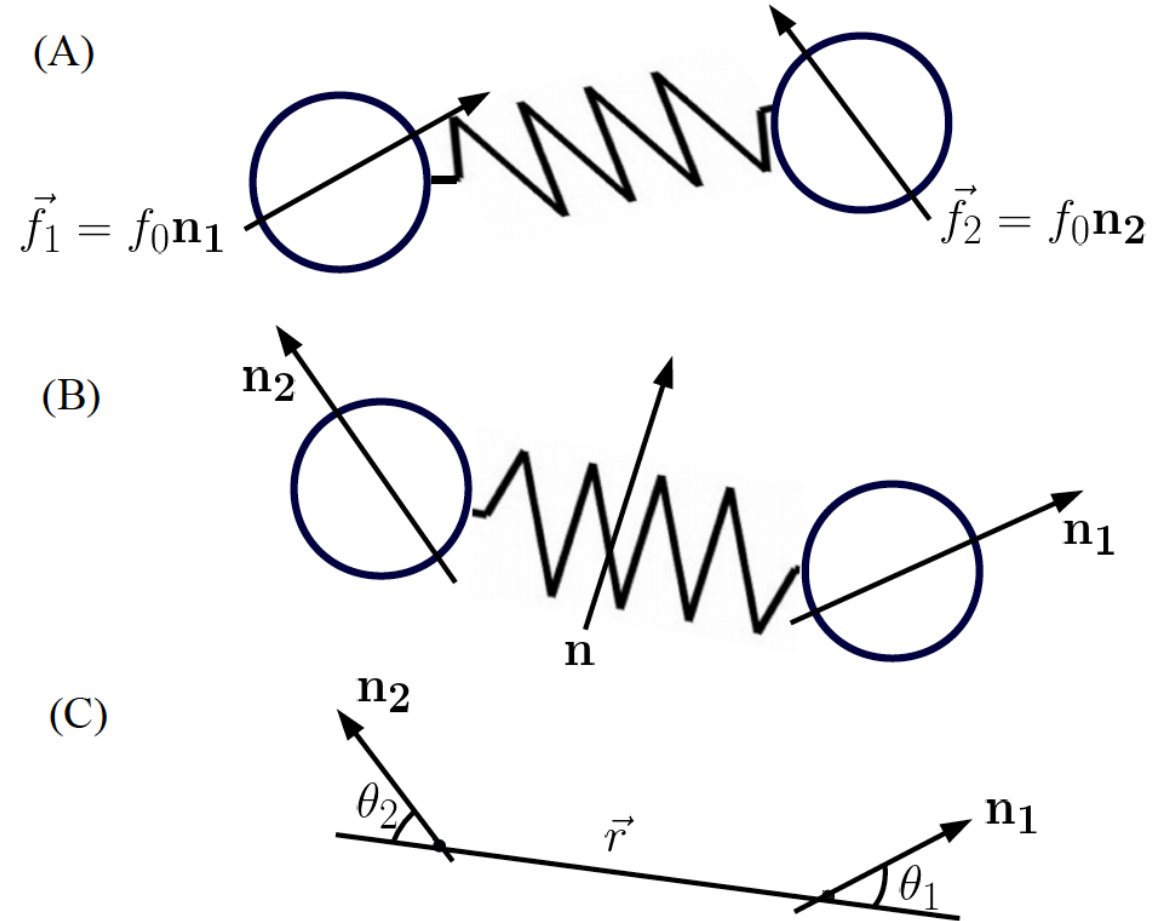
- Stress is **positive**, hence **tensile** in nature.

**Fluidity → Outward alignment of active force  
→ Tensile stress**



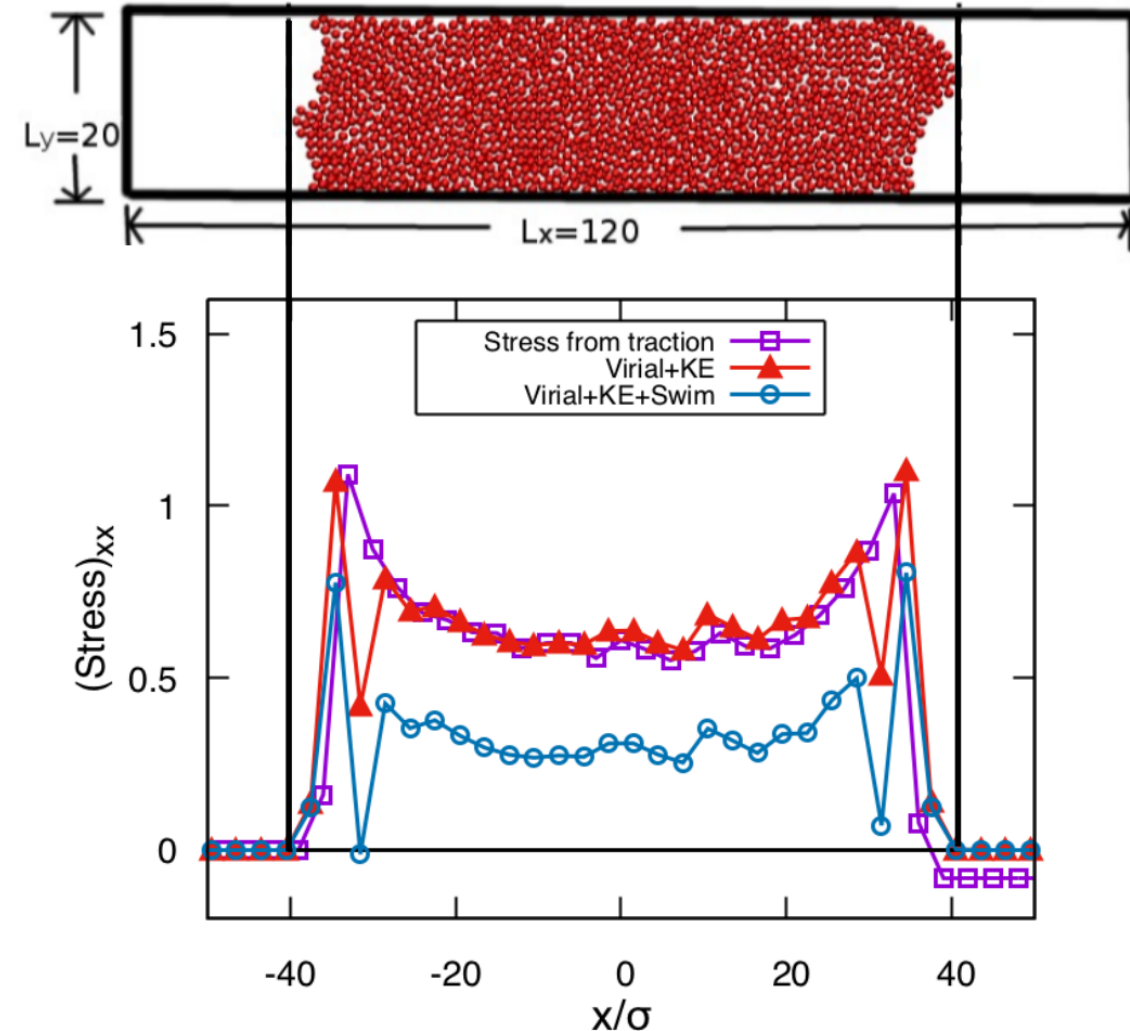
# THE ORIGIN OF TENSION

- Two ABPs + Spring
- Simple all-state average yields  
Average Tension:  
$$\langle f_{ext} \rangle = \frac{2f_0}{\pi}$$
- Origin: Dumbbells aligne such that the radial force component points outward



# A NOTE ON STRESS:

- Definition of Stress in ABP-Systems is non-unique in the literature
- Disagreement on role of active contributions
- Passive contributions (KE + Virial) clear
- The underlying physical system is important:  
Here a ***cell crawling on a substrate!***
- We discuss the stress in the cell layer  
=> Force balance can define the stress

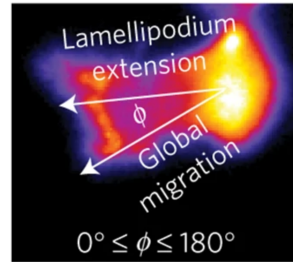


# VELOCITY ALIGNMENT INTERACTIONS

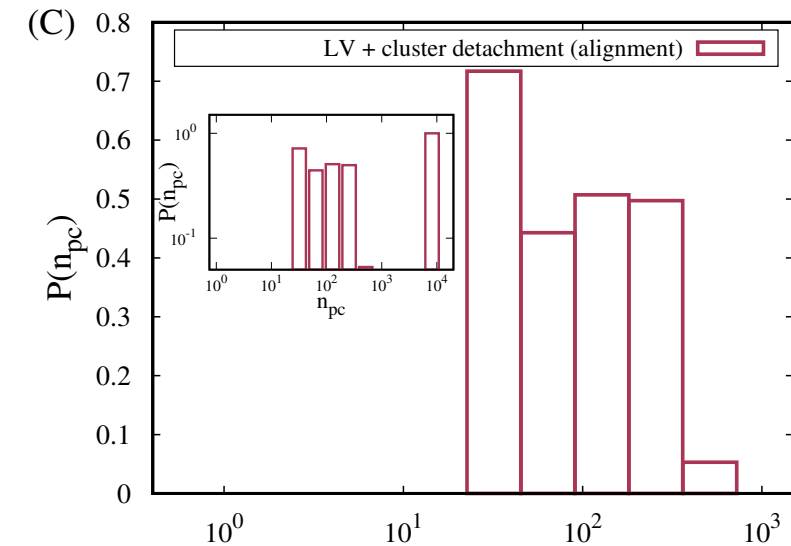
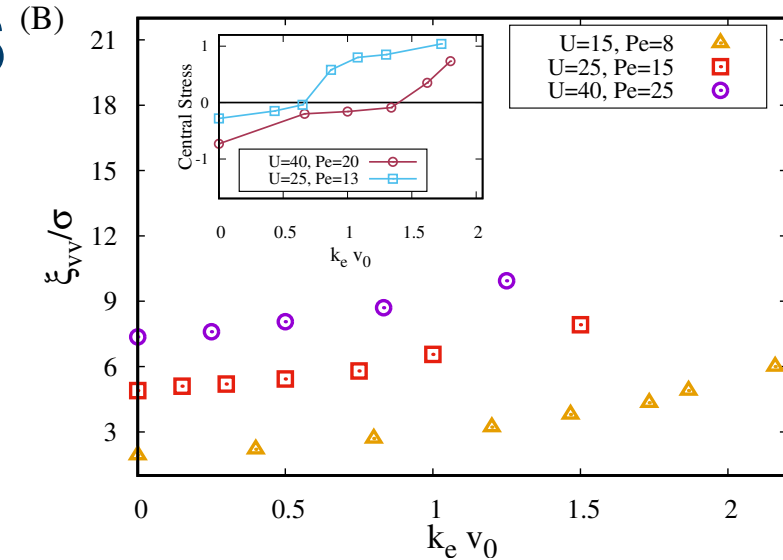
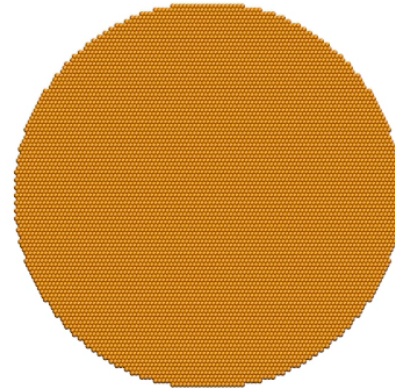
- Alignment of direction with velocity

$$\dot{\theta}_i = \sqrt{2D_r}\eta_i^R - k_e D_r \frac{\partial}{\partial \theta} (\mathbf{n}_i \cdot \mathbf{v}_i)$$

- Enhances effects, and leads to more "cell like" behavior
- Longer correlation lengths
- Larger clusters detaching
- More tension

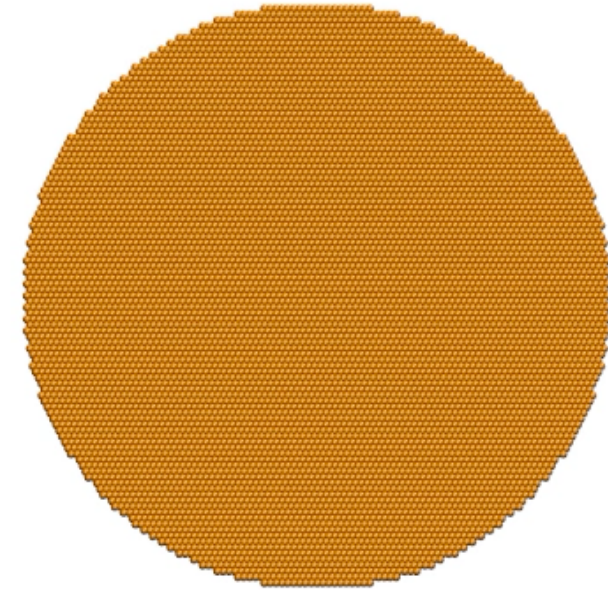


compare talk from  
Roberto Cerbino  
yesterday  
Malinverno et al. Nat. Mat. (2017)



# SECOND CONCLUSIONS

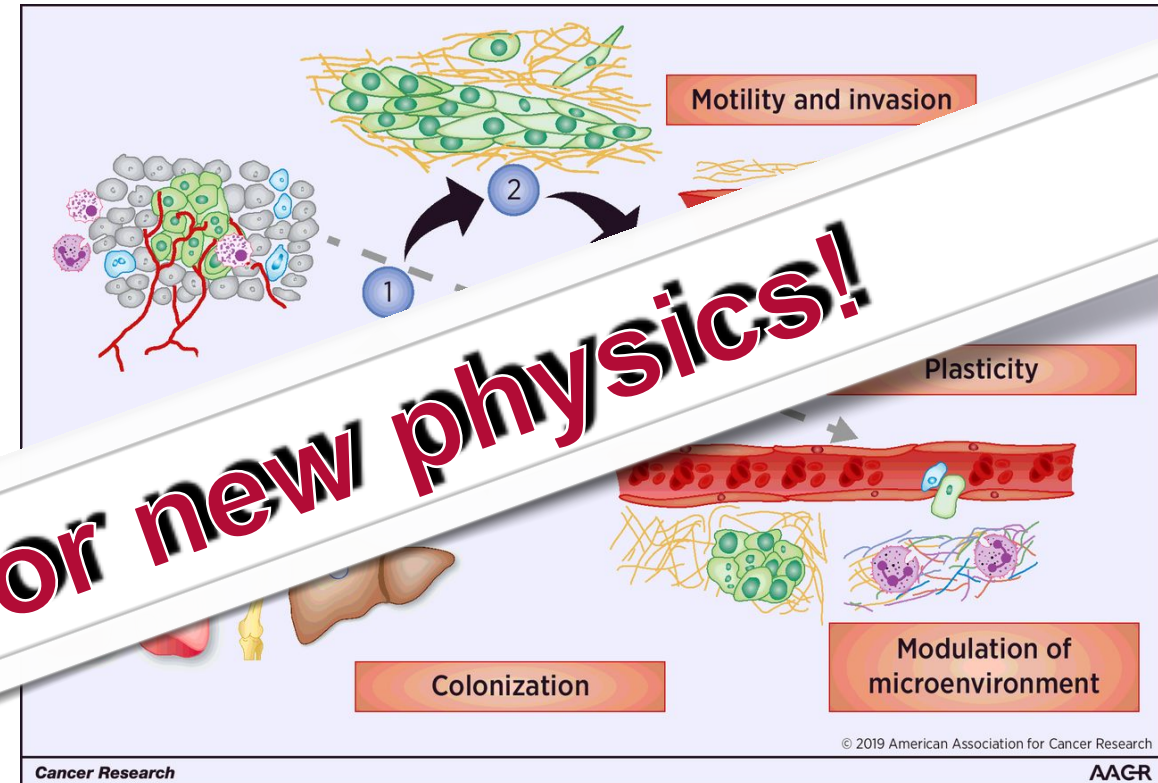
- Liquid-"Vacuum" coexistence in a simple ABP Model
- Tension arises naturally
- "Cells" detach in clusters
- Velocity alignment enhances these effects



# THE BIG QUESTION

- What happens if we combine growth and motility?
- Does motility induced tension help growth?
- Can a "cost" of motility be over-compensated due to mechanical advantages?
- How does the meta evolve?

• **Lots of room for new physics!**



Welch and Hurst (2019). Cancer Research 79, 12

# THANKS

## Support

- IBI-5: Theoretical Physics of Living Matter
- Computing Time: JARA-HPC VSR-Grant jics23 & jiff26
- Priority Programme SPP 1726
- Helmholtz-Society/FZJ

**Let's talk!!!**

Jens@elgeti.de



## The Team

- |                   |                        |
|-------------------|------------------------|
| • Sebastian Rode  | • Nils Podewitz        |
| • Raphael Hornung | • Guglielmo Saggiorato |
| • Lucas Campos    | • Nirmalendu Ganai     |
| • Özer Duman      | • Rolf Isele-Holder    |
| • Tobias Büscher  | • Debarati Sarkar      |