Shadow Momenta and New Constraints on Celestial Amplitudes

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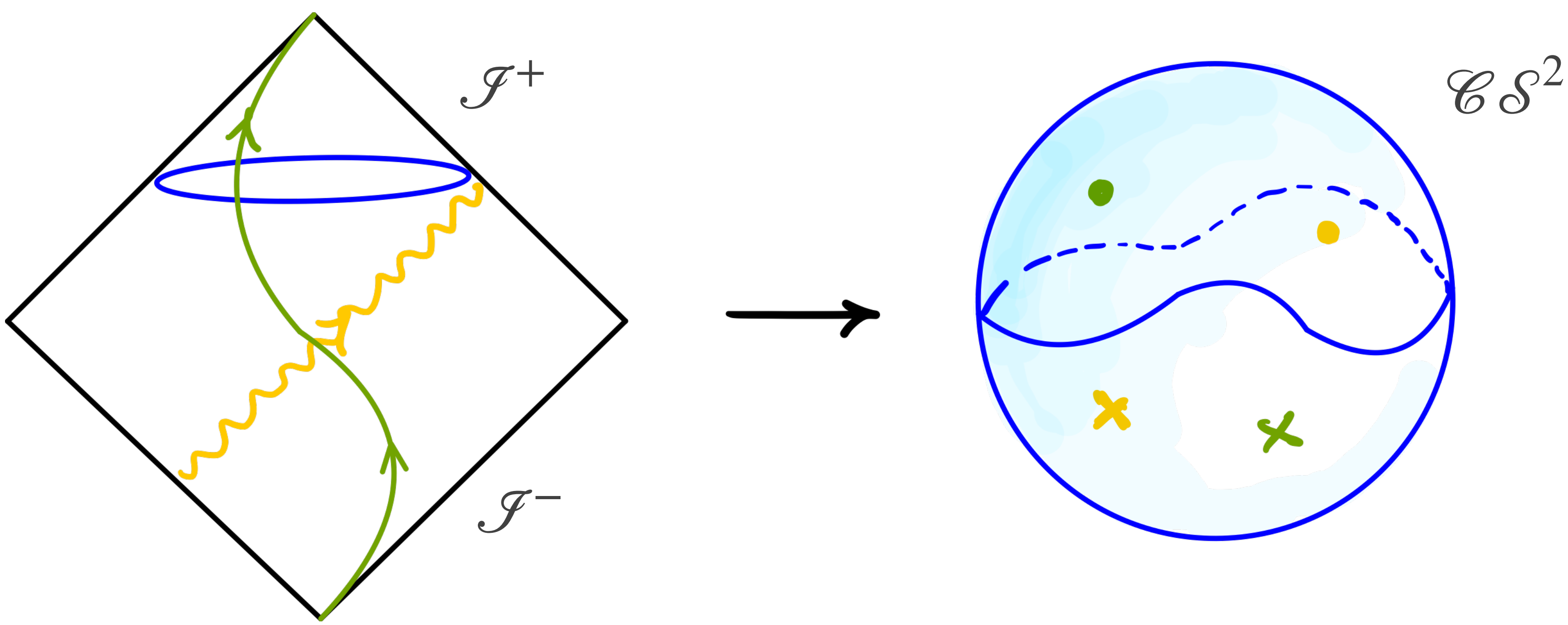
Based on work in progress with Laurent Freidel

Celestial holography

- New framework for studying scattering in asymptotically flat space
- Celestial sphere: natural space to define scattering observables



- Celestial amplitude = S-matrix in basis of boost eigenstates

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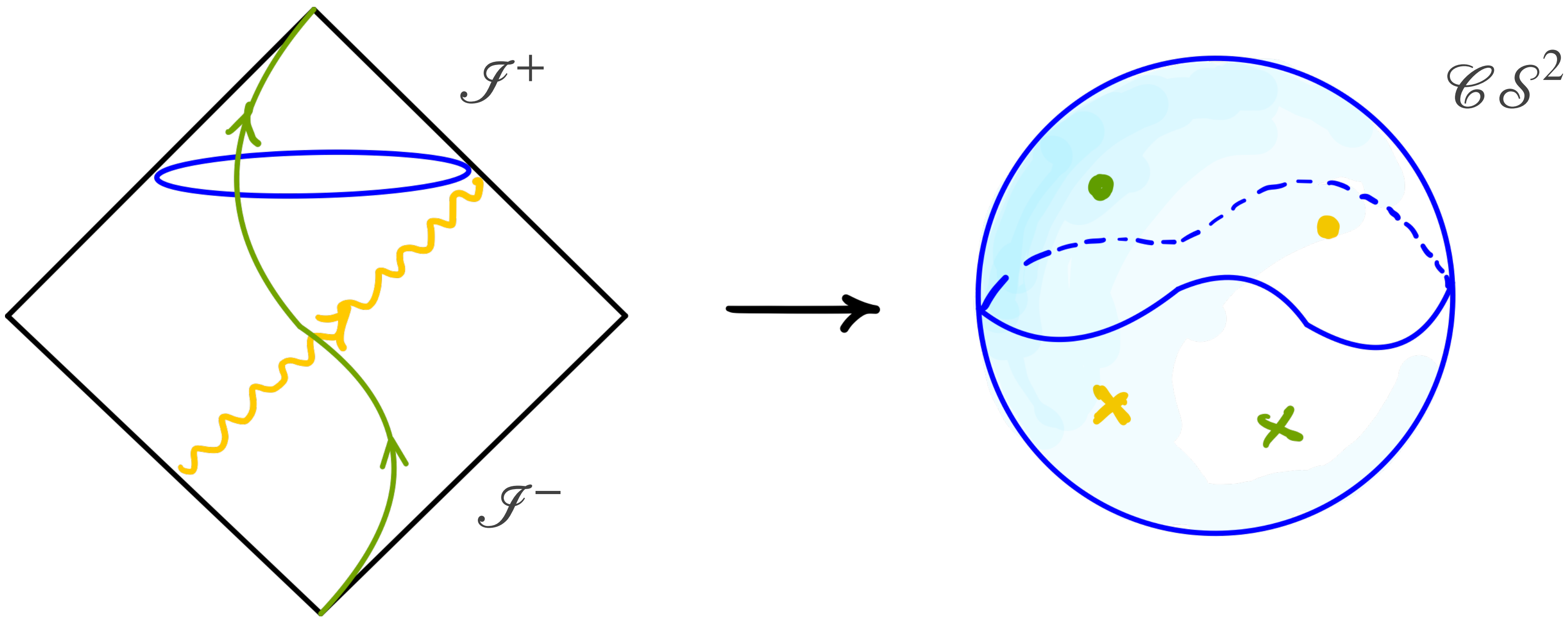

- Massless states $|p_i\rangle \to |\omega_i, z_i, \bar{z}_i\rangle \to |\Delta_i, z_i, \bar{z}_i\rangle = \int_0^\infty d\omega_i \omega_i^{\Delta_i - 1} |\omega_i, z_i, \bar{z}_i\rangle$

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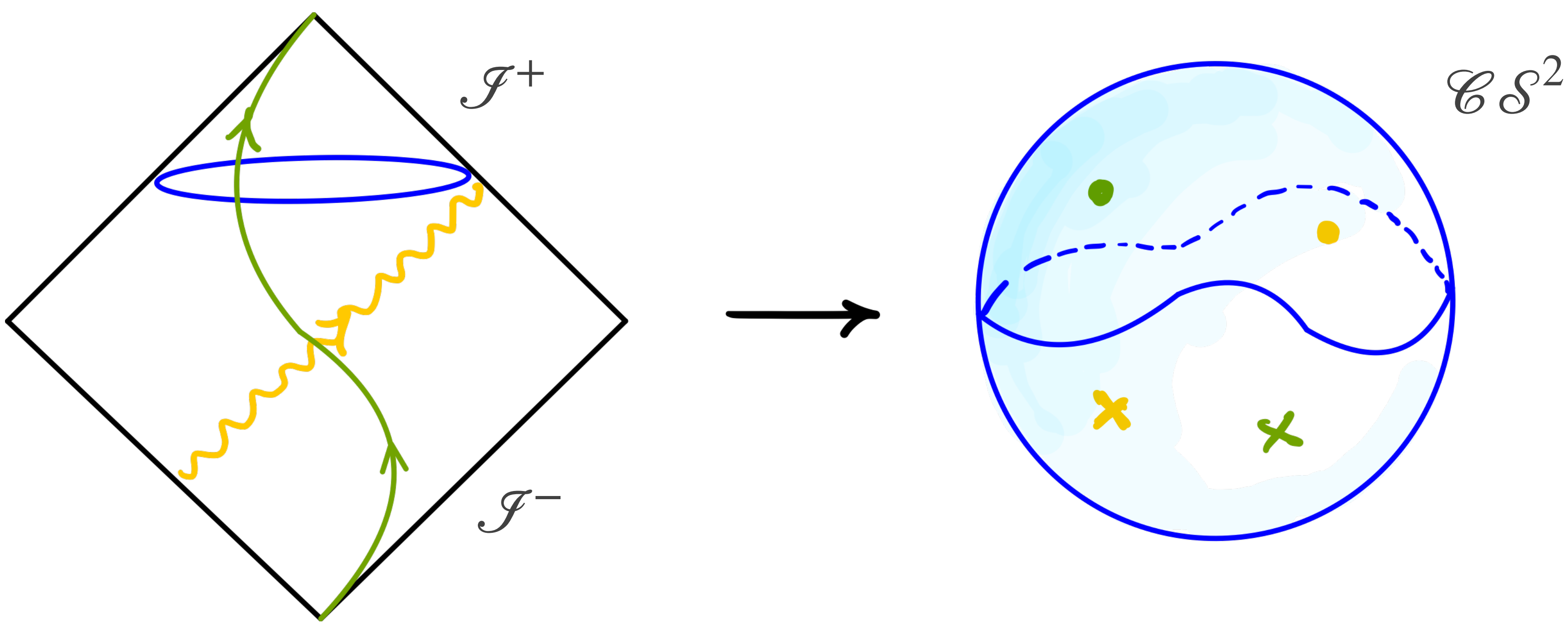


- Many connections: symmetries, soft theorems, CFT, scattering amplitudes, twistors,...

Outline

- Celestial amplitudes: properties, symmetries, constraints
- Shadow/light-transforms
- Revisiting the celestial Poincare algebra: momenta and their shadows
- Constraints from shadow symmetries

Celestial amplitudes

- Massless scalar scattering characterized by a function of n on-shell momenta:

$$p_i = \epsilon_i \omega_i \left(1 + z_i \bar{z}_i, z_i + \bar{z}_i, -i(z_i - \bar{z}_i), 1 - z_i \bar{z}_i\right).$$

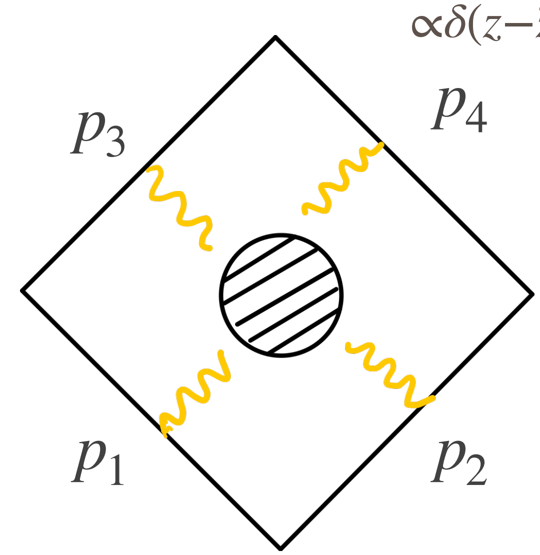
- Map from momentum space to celestial amplitude:

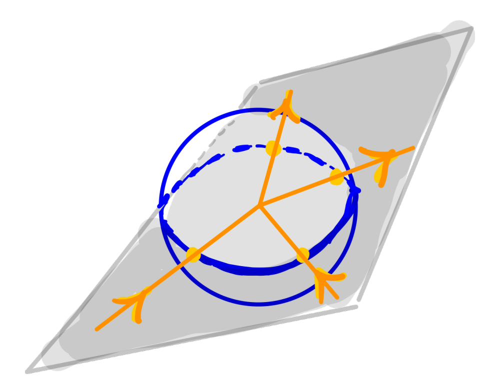
$$\widetilde{\mathscr{A}}(\Delta_1, z_1, \bar{z}_1; \cdots) = \prod_{i=1}^{n} \int_0^\infty d\omega_i \omega_i^{\Delta_i - 1} \mathscr{A}(\omega_1, z_1, \bar{z}_1; \cdots).$$

- Example: 4-point scattering $\widetilde{\mathscr{A}}(z_i, \bar{z}_i; \beta) = K(z_i, \bar{z}_i) \underbrace{X(z, \beta)}_{\propto \delta(z - \bar{z})} \int_0^\infty d\omega \omega^{\beta - 1} \mathscr{M}(\omega^2, -z\omega^2).$

$$s = -(p_1 + p_2)^2 \to \beta = \sum_{i=1}^{4} (\Delta_i - 1)$$

$$t = -(p_1 + p_3)^2 \to z = -t/s = \frac{z_{13} z_{24}}{z_{12} z_{34}}$$





Symmetries & constraints

- Example: massless 4-point $\widetilde{\mathscr{A}}(z_i,\bar{z}_i;\beta) = K(z_i,\bar{z}_i)\underbrace{X(z,\beta)}_{\propto\delta(z-\bar{z})}\int_0^\infty d\omega\,\omega^{\beta-1}\mathscr{M}(\omega^2,-z\omega^2)\,.$
- $K(z_i,\bar{z}_i)$ fixed by Lorentz SL(2,ℂ)

$$\sum_{N=1}^{4} L_i^{(N)}\widetilde{\mathscr{A}}(\Delta_i,z_i,\bar{z}_i) = 0, \quad \begin{cases} L_0^{(i)} = 2(z_i\partial_{z_i}+h_i), & L_-^{(i)} = \partial_{z_i}, \quad L_+^{(i)} = z_i^2\partial_{z_i}+2z_ih_i, \\ \bar{L}_0^{(i)} = 2(\bar{z}_i\partial_{\bar{z}_i}+\bar{h}_i), & \bar{L}_-^{(i)} = \partial_{\bar{z}_i}, \quad \bar{L}_+^{(i)} = \bar{z}_i^2\partial_{\bar{z}_i}+2\bar{z}_i\bar{h}_i\,. \end{cases}$$

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- SL(2,$\mathbb{C}$) & momentum conservation

$$\sum_{N=1}^{4} P^{(N)} \widetilde{\mathscr{A}}(\Delta_i, z_i, \bar{z}_i) = 0, \qquad P\mathcal{O}_\Delta(z_i, \bar{z}_i) = \epsilon_i \mathcal{O}_{\Delta+1}(z_i, \bar{z}_i) \implies$$

remaining dependence on z and $\beta = \sum_{i=1}^{4} \Delta_i - 4$ only.

Symmetries & constraints

- Massive scalars $P_\mu^{(m)} = \frac{m}{2}\left[\left(\partial_z\partial_{\bar z}\hat q_\mu + \frac{\partial_{\bar z}\hat q_\mu\partial_z + \partial_z\hat q_\mu\partial_{\bar z}}{\Delta-1} + \frac{\hat q_\mu\partial_z\partial_{\bar z}}{(\Delta-1)^2}\right)e^{-\partial_\Delta} + \frac{\Delta\hat q_\mu}{\Delta-1}e^{\partial_\Delta}\right]$

[Law, Zlotnikov '19,'20]

- Lead to intricate set of constraints for 4-point massive celestial amplitudes.

- Simple story for 3-point amplitudes of 2 massless and one massive scalars:

$$\widetilde{\mathscr{A}}(1,2,3^{(m)}) = \frac{C(\Delta_1,\Delta_2,\Delta_3)}{|z_{12}|^{\Delta_1+\Delta_2-\Delta_3}|z_{23}|^{\Delta_2+\Delta_3-\Delta_1}|z_{13}|^{\Delta_1+\Delta_3-\Delta_2}},$$

$$\left(P_1+P_2+P_3^{(m)}\right)\widetilde{\mathscr{A}}_3(1,2,3^{(m)}) = 0 \implies C(\Delta_1,\Delta_2,\Delta_3) \propto B\left(\frac{\Delta_{12}+\Delta_3}{2},\frac{-\Delta_{12}+\Delta_3}{2}\right).$$

- Infinity of soft currents, symmetry algebra

[Banerjee, Ghosh, Samal '20,…, Guevara, Himwich, Pate, Strominger '21, Strominger '21, Himwich, Pate, Singh '21]

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A puzzle

- In theories with massless particles only, 2 and 3-point functions are singular.
- Celestial gluon three-point function

$$\widetilde{\mathscr{A}}_3 \equiv \langle \mathcal{O}_1^-(z_1,\bar{z}_1)\mathcal{O}_2^-(z_2,\bar{z}_2)\mathcal{O}_3^+(z_3,\bar{z}_3)\rangle \propto \delta(\lambda_1+\lambda_2+\lambda_3) z_{12}^{1-i(\lambda_1+\lambda_2)} z_{23}^{i\lambda_1-1} z_{13}^{i\lambda_2-1}\delta(\bar{z}_{13})\delta(\bar{z}_{23}),$$

$$??? \implies \widetilde{\mathscr{A}}_3(--+) \propto \frac{1}{z_{12}^{h_1+h_2-h_3} z_{23}^{h_2+h_3-h_1} z_{13}^{h_1+h_3-h_2}} \times (z \to \bar{z}, h \to \bar{h})\,.$$

[Sharma '21]

- Celestial two-point function $\langle \mathcal{O}_1(z_1,\bar{z}_1)\mathcal{O}_2(z_2,\bar{z}_2)\rangle \propto \delta(\Delta_1+\Delta_2-2)\delta^{(2)}(z_1-z_2)$

  becomes a standard CFT 2-point function upon defining a new inner product.

[Pasterski, Shao '17; Crawley, Miller, Narayanan, Strominger '21]

$$\big(\Phi_1,\Phi_2\big)_S = i(\widetilde{\Phi}_1,\Phi_2^*)_{KG} \implies \langle \mathcal{O}_1(z_1,\bar{z}_1)\widetilde{\mathcal{O}}_2(z_2,\bar{z}_2)\rangle \propto \frac{\delta(\Delta_1-\Delta_2)}{|z_1-z_2|^{2\Delta_1}}, \quad \Delta_i = 1 + i\lambda_i\,.$$

Shadow- and light-transforms

- Shadow transform $\mathcal{S}[\mathcal{O}]_{1-h,1-\bar{h}}(z,\bar{z}) = N_{h,\bar{h}} \int d^2w \frac{1}{(z-w)^{2-2h}} \frac{1}{(\bar{z}-\bar{w})^{2-2\bar{h}}} \mathcal{O}_{h,\bar{h}}(w,\bar{w})$

  maps conformal primaries of (Δ, J) to primaries of $(2-\Delta, -J)$.

$$\mathcal{S}^2 = (-1)^{2(h-\bar{h})} \implies 16\pi^2 N_{(1-h,1-\bar{h})} N_{(h,\bar{h})} \frac{\Gamma(1-2\bar{h})\Gamma(2\bar{h}-1)}{\Gamma(2-2h)\Gamma(2h)} = 1.$$

- Upon shadowing one particle, 4-point kinematic constraint $z = \bar{z}$ is relaxed;

  eg. gluon 4-point functions

$$\widetilde{\mathcal{A}}_4 = K_{h_i,\bar{h}_i}(z_i,\bar{z}_i)\delta(z-\bar{z})f^{\beta,J_i}(z,\bar{z}) \quad \longrightarrow$$

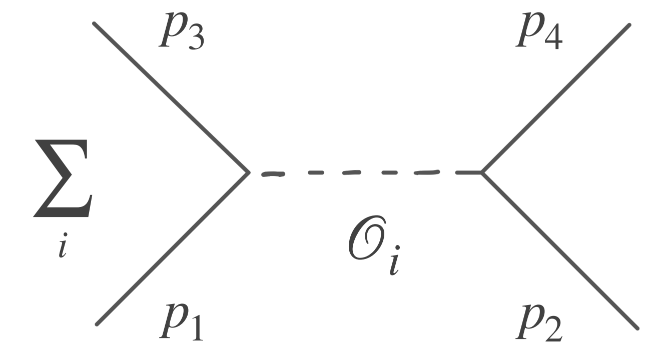
non-singular, admit a conformal block expansion

[Fan, Fotopoulos, Stieberger, Taylor, Zhu '21]

Shadow- and light-transforms

- Light transform $\mathscr{L}[\mathcal{O}]_{1-h,\bar{h}}(z,\bar{z}) = N_h \int dw \frac{1}{(z-w)^{2-2h}} \mathcal{O}_{h,\bar{h}}(w,\bar{w}), \quad (\Delta, J) \to (1-J, 1-\Delta)$

- Appear as exchanges in the conformal block decomposition of four massless scalars [Atanasov, Melton, AR, Strominger '21]



$$\widetilde{\mathscr{A}}_4 = I_{13-24}(z_i,\bar{z}_i) f_t(z,\bar{z}), \quad f_t(z,\bar{z}) = |z|^2 |1-z|^{h_{13}-h_{24}} \delta(z-\bar{z}),$$

$$f_t(z,\bar{z}) = n(\beta) \Big[\sum_{n=1}^{\infty} D^{nn} C_{13n} C_{24n} \cos\pi\left(\frac{n}{2}+h_{13}\right) \cos\pi\left(\frac{n}{2}+h_{24}\right) k_{\frac{1+n}{2}}(z) k_{\frac{1+n}{2}}(\bar{z})$$

$$\underbrace{+\frac{1}{2}\int_{-i\infty}^{i\infty} d\alpha C^L_{13\alpha} C^L_{24\alpha} D^{L,\alpha\alpha} k_{\frac{1}{2}+\alpha}(z) k_{\frac{1}{2}-\alpha}(\bar{z})}_{\text{Light-ray operators}} \Big].$$

- Related to half-Fourier transforms to twistor space. [Sharma '21]

Summary so far and goals

- Massive $$P_\mu^{(m)} = \frac{m}{2}\left[\left(\partial_z\partial_{\bar z}\hat q_\mu + \frac{\partial_{\bar z}\hat q_\mu\partial_z + \partial_z\hat q_\mu\partial_{\bar z}}{\Delta - 1} + \frac{\hat q_\mu\partial_z\partial_{\bar z}}{(\Delta-1)^2}\right)e^{-\partial_\Delta} + \frac{\Delta\hat q_\mu}{\Delta - 1}e^{\partial_\Delta}\right]$$

  symmetry leads to non-singular 3-point functions, so do shadows and light-transforms.

- First principles understanding of celestial Poincare generators for all masses and spins

- Coordinate independent $\longrightarrow$ shadow/light symmetries manifest!

- Constraints on shadow, light transformed celestial amplitudes

Poincare algebra

- Let $z_A, \bar{z}_A$ be spinor helicity variables.

- Introduce conjugate spinors $w_A, \bar{w}_A$such that

$$[w_A, z_B] = \epsilon_{AB}, \quad [\bar{w}_A, \bar{z}_B] = -\epsilon_{AB}, \qquad w_A = -\frac{\partial}{\partial z^A}, \quad \bar{w}_A = \frac{\partial}{\partial \bar{z}^A}\,.$$

- Lorentz generators become

$$L_+ = z_0 w_0, \qquad L_0 = \frac{1}{2}(z_0 w_1 + z_1 w_0), \qquad L_- = z_1 w_1,$$
$$\bar{L}_+ = \bar{w}_0 \bar{z}_0, \qquad \bar{L}_0 = \frac{1}{2}(\bar{w}_0 \bar{z}_1 + \bar{w}_1 \bar{z}_0), \qquad \bar{L}_- = \bar{w}_1 \bar{z}_1\,.$$

- Standard formulas recovered by choosing $z_A \to \begin{pmatrix} 1 \\ z \end{pmatrix} e^{\frac{1}{2}\partial_h}\,.$

Poincare algebra

- Momenta transform as Lorentz vectors

$$[L_+, P_{0A}] = 0, \qquad [L_0, P_{0A}] = -\frac{1}{2}P_{0A}, \qquad [L_-, P_{0A}] = -P_{1A}$$

$$[L_-, P_{1A}] = 0, \qquad [L_0, P_{1A}] = \frac{1}{2}P_{1A}, \qquad [L_+, P_{1A}] = P_{0A}$$

implying the following Ansatz

$$P_{AB} = z_A\alpha(H,\bar{H})\bar{z}_B + z_A\beta(H,\bar{H})\bar{w}_B + w_A\gamma(H,\bar{H})\bar{z}_B + w_A\delta(H,\bar{H})\bar{w}_B\,.$$

- The conformal Casimirs $H(H-1)$, $\bar{H}(\bar{H}-1)$ given in terms of

$$H \equiv \frac{1}{2}z_A w^A, \quad \bar{H} \equiv \frac{1}{2}\bar{z}_B\bar{w}^B, \quad \Delta = H + \bar{H},\ J = H - \bar{H}\,.$$

- Massless case: $[P_{AB}, P_{CD}] = 0$, $\det P = 0 \implies P_{AB} = z_A\bar{z}_B\,.$

[Donnay, Puhm, Strominger '18, Stieberger, Taylor '18]

Poincare algebra

- Massive case: $[P_{AB}, P_{CD}] = 0,$ $\det P = -1$ $\Longrightarrow$

  6 non-trivial recursion relations for $\alpha, \beta, \gamma, \delta$.

- For example $[P_{00}, P_{01}] = 0$ $\Longrightarrow$

$$\bar{H}\alpha(H,\bar{H})\beta\left(H+\frac{1}{2},\bar{H}+\frac{1}{2}\right) - (\bar{H}-1)\alpha\left(H+\frac{1}{2},\bar{H}-\frac{1}{2}\right)\beta(H,\bar{H}) = 0.$$

- Solution:

$$\alpha(H,\bar{H}) = \frac{1}{(2H-1)(2\bar{H}-1)}\widetilde{\alpha}(\Delta), \quad \beta(H,\bar{H}) = \frac{1}{(2H-1)(2\bar{H}-1)}\widetilde{\beta}(J),$$

$$\gamma(H,\bar{H}) = \frac{1}{(2H-1)(2\bar{H}-1)}\widetilde{\gamma}(J), \quad \delta(H,\bar{H}) = \frac{1}{(2H-1)(2\bar{H}-1)}\widetilde{\delta}(\Delta),$$

where $\widetilde{\alpha}, \widetilde{\beta}, \widetilde{\gamma}, \widetilde{\delta}$ obey two non-linear recursion relations.

Massive momenta

- Recursion relations for $\widetilde{\alpha}, \widetilde{\beta}, \widetilde{\gamma}, \widetilde{\delta}$ admit (at least) two families of solutions:

1) $$\widetilde{\alpha}(\Delta-1)=(\Delta+s)(\Delta-1-s), \quad \widetilde{\delta}(\Delta)=-1, \quad \widetilde{\beta}(J)=-J+s, \quad \widetilde{\gamma}(J)=J+s,$$

2) $$\widetilde{\alpha}(\Delta-1)=\Delta-1-s, \quad \widetilde{\delta}(\Delta)=-(\Delta-1)-s, \quad \widetilde{\beta}(J)=-J+s, \quad \widetilde{\gamma}(J)=J+s\,.$$

- 1) unfolds into the expression found by Law, Zlotnikov

$$P^{(m)}_{JI}=\frac{m}{2}\Big(\frac{(J+s)\delta_{J-1I}}{\Delta-1-J}\left(\partial_z\hat{q}+\frac{1}{\Delta+I}\hat{q}\partial_z\right)+\frac{(J-s)\delta_{J+1I}}{\Delta-1+J}\left(\partial_{\bar{z}}\hat{q}+\frac{1}{\Delta-I}\hat{q}\partial_{\bar{z}}\right)$$
$$+\delta_{JI}\left(\partial_z\partial_{\bar{z}}\hat{q}+\frac{\partial_{\bar{z}}\hat{q}\partial_z}{\Delta-1+J}+\frac{\partial_z\hat{q}\partial_{\bar{z}}}{\Delta-1-J}+\frac{\hat{q}\partial_z\partial_{\bar{z}}}{(\Delta-1)^2-J^2}\right)e^{-\partial_\Delta}+\frac{(\Delta-s-1)(\Delta+s)}{(\Delta-1)^2-J^2}\hat{q}e^{\partial_\Delta}\Big)\,.$$

- 2) is new, compactly written as

$$(2H-1)(2\bar{H}-1)P^{(m)}_{AB}=z_A\left(\Delta-\frac{3}{2}-s\right)\bar{z}_A+w_A\left(-\Delta+\frac{1}{2}-s\right)\bar{w}_A+z_A(-J+s)\bar{w}_A+w_A(J+s)\bar{z}_B\,.$$

Poincare shadows

- Under shadow transform $Sz_AS^\dagger = w_A, \;\; S\bar{z}_AS^\dagger = \bar{w}_A\,.$
- Shadow transform preserves the Lorentz generators $SL_nS^\dagger = L_n$

  and the Casimirs, while $SHS^\dagger = 1 - H, \;\; S\bar{H}S^\dagger = 1 - \bar{H}\,.$
- Shadows of massless momenta $\widetilde{P}_{AB} \equiv Sz_A\bar{z}_BS^\dagger = w_A\bar{w}_B \implies \widetilde{P}_{AB} \neq P_{AB}\,.$
- Massive momenta (2) are shadow invariant $\widetilde{P}^{(m)}_{AB} \equiv SP^{(m)}_{AB}S^\dagger = P^{(m)}_{AB}$

  and are associated with shadow invariant conformal primaries.
- Under light transform $Lz_AL^\dagger = w_A, \;\; L\bar{z}_AL^\dagger = \bar{z}_A.$

New constraints

- Shadow and light-transformed momentum generators for massless particles impose constraints on celestial amplitudes:

$$\widetilde{P}_{AB} = w_A \bar{w}_B \rightarrow \frac{(\Delta-1)^2}{2}\left(\partial_z \partial_{\bar{z}} \hat{q} + \frac{1}{\Delta-1}\partial_z \hat{q} \partial_{\bar{z}} + \frac{1}{\Delta-1}\partial_{\bar{z}} \hat{q} \partial_z + \frac{\hat{q}\partial_z \partial_{\bar{z}}}{(\Delta-1)^2}\right) e^{-\partial_\Delta}$$

- Two-point Ansatz $\langle \mathcal{O}_1(z_1,\bar{z}_1)\widetilde{\mathcal{O}}_2(z_2,\bar{z}_2)\rangle = C_2(\Delta_1,\Delta_2)\frac{\delta(\Delta_1-\Delta_2)}{|z_{12}|^{2\Delta_1}}$.

- Shadow momentum conservation $\left(\epsilon_1 P_1 + \epsilon_2 \widetilde{P}_2\right)\langle \mathcal{O}_1(z_1,\bar{z}_1)\widetilde{\mathcal{O}}_2(z_2,\bar{z}_2)\rangle = 0$

  yields the recursion relation $\epsilon_1 \Delta^2 C(\Delta,\Delta) - \epsilon_2 C(\Delta+1,\Delta+1) = 0.$

Solutions: $C(\Delta_1,\Delta_2) \propto \Gamma(\Delta_1)\Gamma(\Delta_2),\quad \epsilon_1 = \epsilon_2,\quad C(\Delta_1,\Delta_2) \propto \frac{\Gamma(\Delta_2)}{\Gamma(1-\Delta_1)},\quad \epsilon_1 = -\epsilon_2\,.$

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New constraints

Apply similar logic to three-point functions:

- One shadow: constraints inconsistent $\Longrightarrow$ no non-singular solution
- Two and three shadow constraints admit solutions consistent with direct computations
- Appears to work for arbitrary spins (still checking).
- Celestial amplitudes well behaved in shadow-symmetric bases $\Longrightarrow$ shadow symmetry: organizing principle for CCFT?

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- New basis in which 3-point functions are non-singular?

- Role of shadow- and light-transforms in CCFT: enhanced symmetry algebra?