FROM DIAMONDS TO PYRAMIDS ON THE CELESTIAL SPHERE

ANDREA PUHM

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Celestial holography

attempts to apply holographic principle to asymptotically flat spacetimes.

What are the operators?

$$ {}_{\text{boost}}\langle out \,|\, \mathcal{S} \,|\, in \rangle_{\text{boost}} = \langle \mathcal{O}^{\pm}_{\Delta_1,J_1}(w_1,\bar{w}_1)\ldots\mathcal{O}^{\pm}_{\Delta_n,J_n}(w_n,\bar{w}_n)\rangle_{\text{celestial CFT}} $$

What is this CFT?

What is the organizing principle?

Can we bootstrap the conformally soft sector?

Can we bootstrap quantum gravity in asymptotically flat space?

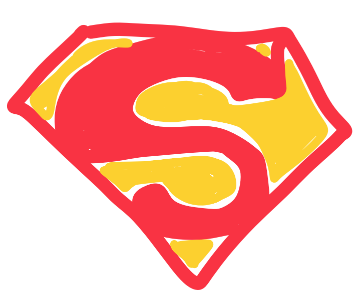
Celestial holography

Key insight: asymptotic symmetries not only represent infinite dimensional enhancement of the familiar global symmetries, but are also realised perturbatively in quantum field theory via soft theorems.

UV/IR mixing of bulk observables ⇒ interesting room for string theory.

Real power: pushing the correspondence to full quantum theory of gravity.

⇒ Supersymmetry and infinite-dimensional fermionic symmetries.



Plan of the talk

BASED ON

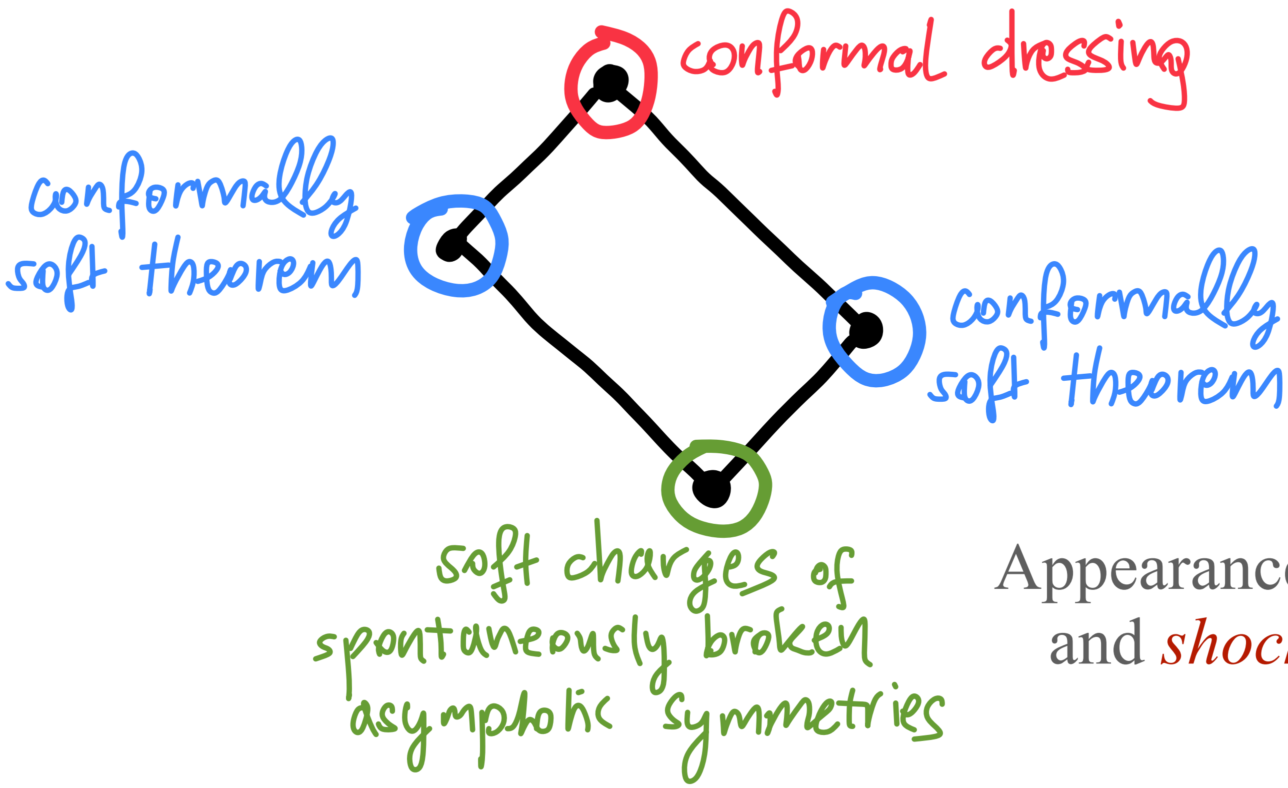
2012.15694 - SABRINA PASTERSKI

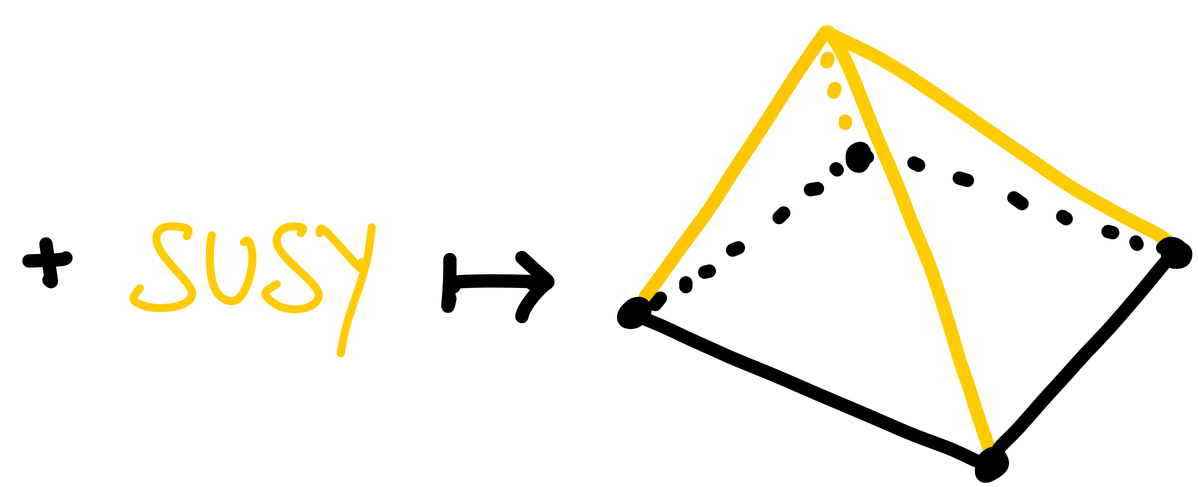
2105.03516 & 2105.09792 - SABRINA PASTERSKI & EMILIO TREVISANI

↦ 2108.11422 - YORGO PANO & SABRINA PASTERSKI

CELESTIAL DIAMONDS & PYRAMIDS

Natural structure in celestial CFT that unifies the discussion of ***soft sector****:*





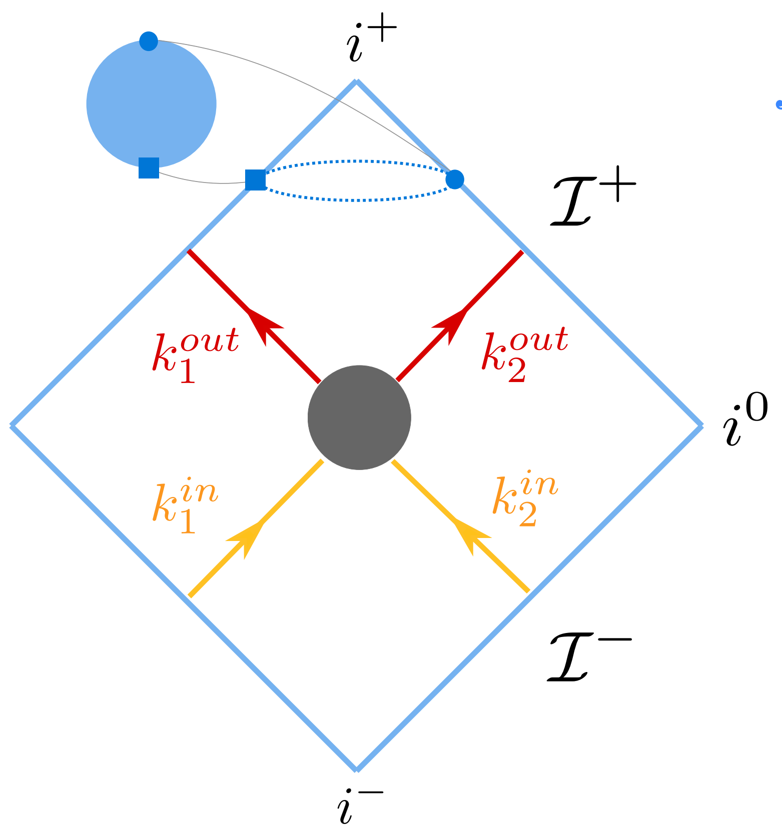
Appearance of *celestial double copy* and *shockwaves* in the diamond.

global SL(2,ℂ) multiplets

4D $\mathcal{S}$-matrix ⇒ 2D correlator

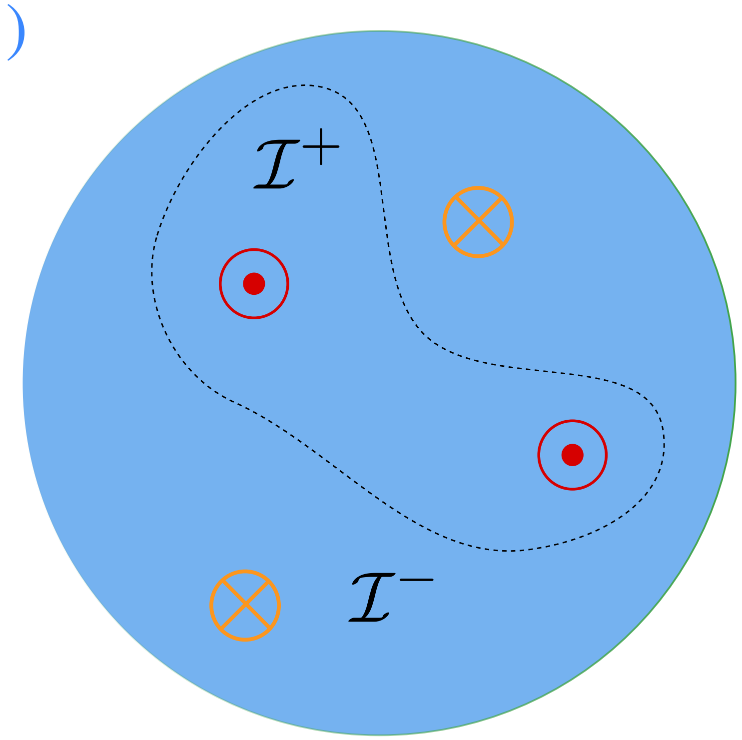
[de Boer,Solodhukin'03] [Cheung,de la Fuente,Sundrum'16][Pasterski,Shao,Strominger'16]

For massless scattering the map is a Mellin transform:



$$\mathcal{M}(\,.\,) = \int_0^\infty \frac{d\omega}{\omega}\omega^\Delta(\,.\,)$$

$$=$$



$$\mathcal{A}_n(\{k_i^\mu = \omega_i q^\mu(w_i, \bar{w}_i); \ell_i\}) \qquad \langle \prod_{i=1}^{n} \mathcal{O}_{\Delta_i, J_i}(w_i, \bar{w}_i) \rangle$$

energy, helicity, point on S^2 — conformal dimension, spin, point on S^2

Lorentz symmetry — conformal symmetry

Conformal Primary Operators

Given a 4D operator $\hat{O}^s(X^\mu)$ of spin-s in the Heisenberg picture, we can define a 2D operator $\mathcal{O}^{s,\pm}_{\Delta,J}(w,\bar{w})$ in the celestial CFT via a suitable inner product with *conformal primay wavefunctions* $\Phi^s_{\Delta,J}(X^\mu_\mp;w,\bar{w})$.

2D celestial CFT operator

4D wavefunction

$$\mathcal{O}^{s,\pm}_{\Delta,J}(w,\bar{w}) \equiv i\left(\hat{O}^s(X^\mu),\Phi^s_{\Delta^*,-J}(X^\mu_\mp;w,\bar{w})\right)_\Sigma$$

4D bulk operator

expanded into modes

inner product = integral over Cauchy slice Σ in the 4D bulk

[Donnay,AP,Strominger'18]
[Donnay,Pasterski,AP'20]
[Pasterski,AP,Trevisani'21]

The $\pm$ label indicates *in* vs *out* states selected by prescription for analytically continuing the wavefunctions as $X^\mu_\pm = X^\mu \pm i\varepsilon\{-1,0,0,0\}$.

Wavefunction based approach connects 2D celestial CFT states to 4D bulk physics via direct construction of celestial operators.

Conformal Primary Wavefunctions

$$\Phi^{s=|J|}_{\Delta,J}(X^\mu; w, \bar{w}) \simeq \int_0^\infty \frac{d\omega}{\omega}\omega^\Delta \epsilon_{\mu_1 \ldots \mu_s} e^{\pm i\omega q\cdot X}$$

4D spin-s field under Lorentz transformations

2D conformal primary with conformal dimension Δ and spin J

	Lorentz transformation		conformal transformation
bulk point	$X^\mu \mapsto \Lambda^\mu{}_\nu X^\nu$	boundary point	$w \mapsto \dfrac{aw+b}{cw+d} \quad \bar{w} \mapsto \dfrac{\bar{a}\bar{w}+\bar{b}}{\bar{c}\bar{w}+\bar{d}}$

$ad - bc = 1 = \bar{a}\bar{d} - \bar{b}\bar{c}$

$$\Phi^s_{\Delta,J}\left(\Lambda^\mu{}_\nu X^\nu; \frac{aw+b}{cw+d}, \frac{\bar{a}\bar{w}+\bar{b}}{\bar{c}\bar{w}+\bar{d}}\right) = (cw+d)^{\Delta+J}(\bar{c}\bar{w}+\bar{d})^{\Delta-J} D_s(\Lambda)\Phi^s_{\Delta,J}(X^\mu; w, \bar{w})$$

3+1D spin-s representation of the Lorentz algebra

[Pasterski,Shao'17]

Radiative: $J = \pm s$ & solve the linearized eom for massless spin-s particles

Generalized: $|J| \leq s$ & allow sources and distributions [Pasterski,AP'20]

Celestial amplitudes

[Pasterski,Shao,Strominger'16+'17]

$\mathcal{S}$-matrix elements constructed as

$$\widetilde{\mathcal{A}}(\Delta_i, z_i, \bar{z}_i) = \prod_{i=1}^{n} \int_0^\infty \frac{d\omega_i}{\omega_i} \omega_i^{\Delta_i} \mathcal{A}(\omega_i, z_i, \bar{z}_i)$$

transform by construction as correlators of (quasi)-primaries

$J_j = j$-th helicity

$$\widetilde{\mathcal{A}}\left(\Delta_i, \frac{az_i + b}{cz_i + d}, \frac{\bar{a}\bar{z}_i + \bar{b}}{\bar{c}\bar{z}_i + \bar{d}}\right) = \prod_{j=1}^{n} (cz_j + d)^{\Delta_j + J_j} (\bar{c}\bar{z}_j + \bar{d})^{\Delta_j - J_j} \widetilde{\mathcal{A}}(\Delta_i, z_i, \bar{z}_i) \,.$$

since the external particles are in boost eigenstates.

Spectrum vs Symmetries

The transformation can be inverted easily when Δ is on the principal continuous series of the Lorentz group $\Delta = 1 + i\mathbb{R}$ which captures finite energy radiation:

$$\mathscr{A}(\omega_i, z_i, \bar{z}_i) = \prod_{i=1}^{n} \int_{1-i\infty}^{1+i\infty} \frac{d\Delta_i}{2\pi i} \omega_i^{-\Delta_i} \widetilde{\mathscr{A}}(\Delta_i, z_i, \bar{z}_i) \,.$$

Analytic continuation to $\Delta \in \mathbb{C}$ of great interest to celestial CFT.

[Donnay,Pasterski,AP'20]

Translations shift the conformal dimension:

$$\mathscr{P}^\mu = q^\mu e^{\partial_\Delta} \Leftrightarrow \Delta \mapsto \Delta + 1$$

[Donnay,AP,Strominger'18]
[Stieberger,Taylor'18]

Takes finite energy $\Delta = 1 + i\mathbb{R}$ away from the principal series.
Shifts conformally soft $\Delta \in \frac{1}{2}\mathbb{Z}$ by integer steps.

Building blocks: null tetrad and spin frame

From $\{X^\mu, q^\mu, \underbrace{\epsilon^\mu_w, \epsilon^\mu_{\bar{w}}}\}$ construct null tetrad for Minkowski

[Pasterski,AP'20]

$$\hookrightarrow \sqrt{2}\epsilon^\mu_\alpha = \partial_\alpha q^\mu$$

$$q^\mu = (1 + w\bar{w}, w + \bar{w}, -i(w - \bar{w}), 1 - w\bar{w})$$

$\Phi^{s\in\mathbb{Z}}_{\Delta,J}$:

$$l^\mu = \frac{q^\mu}{-q\cdot X} \qquad m^\mu = \epsilon^\mu_w + (\epsilon_w \cdot X)l^\mu$$

$$n^\mu = X^\mu + \frac{X^2}{2}l^\mu \qquad \bar{m}^\mu = \epsilon^\mu_{\bar{w}} + (\epsilon_{\bar{w}} \cdot X)l^\mu$$

w/ $l \cdot n = -1$, $m \cdot \bar{m} = 1$

$\Phi^{s\in\frac{1}{2}\mathbb{Z}}_{\Delta,J}$:

$$l_{\alpha\dot{\beta}} = o_\alpha \bar{o}_{\dot{\beta}} \qquad n_{\alpha\dot{\beta}} = \iota_\alpha \bar{\iota}_{\dot{\beta}} \qquad m_{\alpha\dot{\beta}} = o_\alpha \bar{\iota}_{\dot{\beta}} \qquad \bar{m}_{\alpha\dot{\beta}} = \iota_\alpha \bar{o}_{\dot{\beta}}$$

	l^μ	n^μ	m^μ	$\bar{m}^\mu$	o_α	$\bar{o}_{\dot{\alpha}}$	ι_α	$\bar{\iota}_{\dot{\alpha}}$
Δ	0	0	0	0	0	0	0	0
J	0	0	$+1$	-1	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$

$SL(2,\mathbb{C})$ quantum numbers of the tetrad and spin frame

Building blocks: spin-s wavefunctions

[Pasterski,Shao'17]
[Pasterski,AP'20]

Embedding S^2 into $\mathbb{R}^{1,3}$ lightcone: $q^\mu = (1 + w\bar{w}, w + \bar{w}, -i(w - \bar{w}), 1 - w\bar{w})$

$$\sqrt{2}\epsilon_+^\mu = \partial_w q^\mu, \sqrt{2}\epsilon_-^\mu = \partial_{\bar{w}} q^\mu$$

Mellin transform of plane waves :

$$\mathscr{M}(e^{\pm ik\cdot X}) = \int_0^\infty \frac{d\omega}{\omega}\omega^\Delta e^{\pm i\omega q\cdot X_\pm} = \frac{(\mp i)^\Delta \Gamma(\Delta)}{(-q\cdot X_\pm)^\Delta}$$

&

null tetrad

$$\{\underbrace{l, n}_{J=0}, \underbrace{m, \bar{m}}_{J=\pm 1}\}$$

spin frame

$$\{\underbrace{o, \bar{o}, \iota, \bar{\iota}}_{J=\pm\frac{1}{2}}\}$$

Spin $s = 0$:

$$\varphi^\Delta = \frac{1}{(-q\cdot X)^\Delta}$$

Spin $s = \frac{1}{2}, 1, \frac{3}{2}, 2$:

$$\psi_{\Delta, J=+\frac{1}{2}} = o\varphi^\Delta$$

$$A_{\Delta, J=+1} = m\varphi^\Delta$$

$$\chi_{\Delta, J=+\frac{3}{2}} = om\varphi^\Delta$$

$$h_{\Delta, J=+2} = mm\varphi^\Delta$$

$$o_a = \sqrt{q\cdot X}\,|q]_a$$

$$o_a \mapsto (cw + d)^{\frac{1}{2}}(\bar{c}\bar{w} + \bar{d})^{-\frac{1}{2}}(Mo)_a$$

$$m^\mu = \epsilon_+^\mu + (\epsilon_+ \cdot X) l^\mu$$

spacetime dependent polarization vector

$$m^\mu \mapsto \frac{cw + d}{\bar{c}\bar{w} + \bar{d}}\Lambda^\mu{}_\nu m^\nu$$

Spin-s via double copy and supersymmetry

Supersymmetry: relates primaries by $\frac{1}{2}$ integer spin steps

$$\mathcal{Q}_a = \partial_\theta |q]_a e^{\partial_\Delta/2} \qquad \bar{\mathcal{Q}}_{\dot{a}} = \theta\langle q|_{\dot{a}} e^{\partial_\Delta/2} \qquad \{\mathcal{Q}_a, \bar{\mathcal{Q}}_{\dot{a}}\} = -\sigma^\mu_{a\dot{a}}\mathcal{P}_\mu \qquad \mathcal{P}_\mu = q_\mu e^{\partial_\Delta}$$

More on this later!

[Fotopoulos,Stieberger,Taylor,Zhu'20]

Kerr-Schild double copy:

$$g_{\mu\nu} = \eta_{\mu\nu} + \underbrace{m_\mu m_\nu \varphi^\Delta}_{h_{\Delta,J=\pm 2;\mu\nu}}$$

[Pasterski,AP'20]

Kerr-Schild vector:

$$g^{\mu\nu}m_\mu m_\nu = 0 \qquad \eta^{\mu\nu}m_\mu m_\nu = 0 \qquad \text{null}$$

$$m^\mu \nabla_\mu m_\nu \propto m_\nu \qquad m^\mu \partial_\mu m_\nu \propto m_\nu \qquad \text{geodesic}$$

[Monteiro,O'Connell,White'14]

$h_{\Delta,J=\pm 2;\mu\nu}$ is *exact* sol to Einstein: Petrov type N

$$R^{\Delta,J}_{\mu\nu} = 0$$

$$\Psi_4 = -\frac{1}{2}\Delta(\Delta-1)\varphi^\Delta$$

$\Delta = 0,1$: asymptotic symmetries!

Finite conformal primary backgrounds

[Pasterski,AP'20]

- Aichelburg-Sexl shockwave or ultraboosted Schwarzschild [Aichelburg,Sexl'71]

$$g_{\mu\nu} = \eta_{\mu\nu} - \underbrace{4G_N \alpha q_\mu q_\nu \log(X^2)\delta(q\cdot X)}_{h^{gen}_{\Delta=-1,J=0;\mu\nu}} \qquad E = \alpha q^0$$

generalized conformal primary metric

- Dray-'t Hooft planar shell [Dray,t'Hooft'86]

$$g_{\mu\nu} = \eta_{\mu\nu} - 4G_N \alpha q_\mu q_\nu \frac{(X^2)}{D-2}\delta(q\cdot X) \qquad \rho = \alpha q^0$$

- Beam-like gravitational wave [Ferrari,Pendenza,Veneziano'88]

$$g_{\mu\nu} = \eta_{\mu\nu} - 4G_N \alpha q_\mu q_\nu f(X^2)\delta(q\cdot X) \qquad E = \alpha q^0 \sqrt{X^2} f'(X^2)$$

- Kerr gyraton or ultraboosted Kerr see [Cristofoli'20] [Arkani-Hamed,Huang,O'Connell'20]...

$$g_{\mu\nu} = \eta_{\mu\nu} - 4G_N \alpha q_\mu q_\nu \log(|X^2 - a^2|)\delta(q\cdot X) \qquad a^\mu = s^\mu/m$$

↦Amplitudes methods to explore non-perturbative bulk physics in celestial CFT.

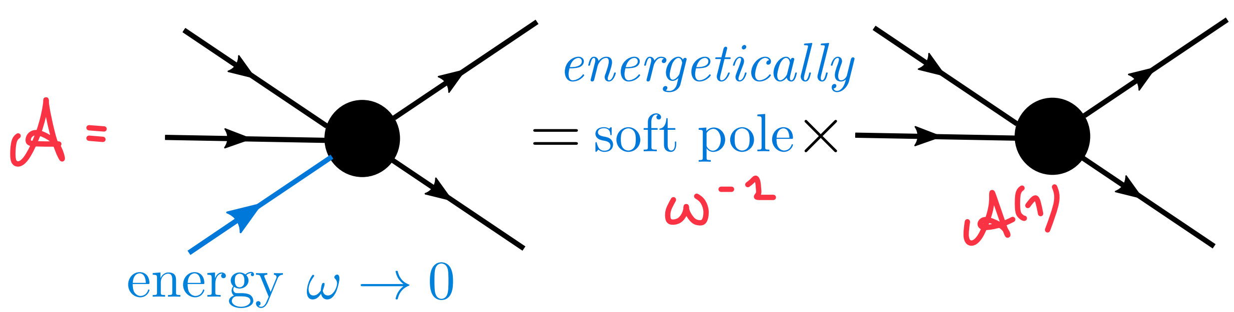
Soft limits as celestial currents

Ward ID of asymptotic symmetries = soft theorems in QFT. [Strominger'13] ...

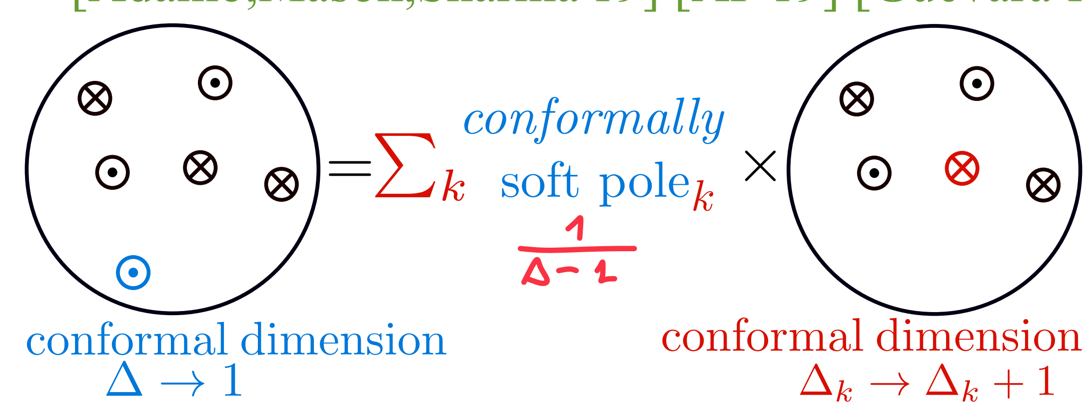
$$\lim_{\omega\to 0}\mathscr{A} = \omega^{-1}\mathscr{A}^{(1)} + \omega^{0}\mathscr{A}^{(0)} + \ldots$$

Soft limits at different orders in $\omega \to 0 \Leftrightarrow$ "conformally soft" Δ poles.

e.g. leading soft graviton: [Weinberg'65]



[Adamo,Mason,Sharma'19] [AP'19] [Guevara'19]



$$\mathscr{P}_w \mathscr{O}_\omega(z,\bar z) \sim \frac{\omega}{w-z}\mathscr{O}_\omega(z,\bar z)$$

$$\mathscr{P}_w \mathscr{O}_\Delta(z,\bar z) \sim \frac{1}{w-z}\mathscr{O}_{\Delta+1}(z,\bar z)$$

[Donnay,AP,Strominger'18]

supertranslation current

Asymptotic symmetry generators

Analytically continuing away from $\Delta = 1 + i\mathbb{R}$ to $\Delta \in \frac{1}{2}\mathbb{Z}$

reveals tip of infinite tower of symmetries:

$$\mathcal{O}_{\Delta,J} \equiv i(\hat{O}, \Phi^{\mathrm{G}}_{\Delta^*,-J})_\Sigma$$

pure gauge

[Donnay,AP,Strominger'18]
[Donnay,Pasterski,AP'20]
[Pasterski,AP'20]
[Pano,Pasterski,AP'21]

$s = \|J\|$	Δ	$\tilde{\Delta}$	energetically soft pole	celestial current	asymptotic symmetry
1	1	1	ω^{-1}	$\mathcal{J}$	large U(1)
$\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\omega^{-\frac{1}{2}}$	$\mathcal{S}$ & $\tilde{\mathcal{S}}$	large SUSY
2	1	1	ω^{-1}	$\mathcal{P}$	supertranslation
	0	2	ω^{0}	$\mathcal{T}$ & $\tilde{\mathcal{T}}$	superrotation

Conformally soft primaries

symmetry generator

$$\mathcal{O}_{\Delta,J} \equiv i(\hat{O}, \Phi^{G}_{\Delta^*,-J})_\Sigma$$

pure gauge

[Donnay,AP,Strominger'18]
[Donnay,Pasterski,AP'20]
[Pasterski,AP'20]
[Pano,Pasterski,AP'21]

	$A_{\Delta,J=\pm 1}$	$\chi_{\Delta,J=+\frac{3}{2}}$	$h_{\Delta,J=\pm 2}$	
Δ	1	$\frac{1}{2}$	1	0
symmetry	large U(1)	large SUSY	supertranslation	shadow superrotation

$\mathcal{J}_w$ $\mathcal{P}_w$ Diff(S^2)

Goldstones of spontaneously broken asymptotic symmetries for $1 \leq s \leq 2$

Kerr-Schild double copy: BMS = (large U(1))^2

[Huang,Kol,O'Connell'19]

	$\psi_{\Delta,J=+\frac{1}{2}}$	$A_{\Delta,J=\pm 1}$	$\chi_{\Delta,J=+\frac{3}{2}}$	$h_{\Delta,J=\pm 2}$
Δ	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1

not pure gauge

[Fotopoulos,Taylor, Stieberger,Zhu'20]
[Adamo,Mason,Sharma'19]
[Guevara,19]

Soft theorems without conformal Goldstones for $1 \leq s \leq 2$

Conformally soft shadow primaries

$$\widetilde{\Phi}_{\Delta,J} \equiv \widetilde{\Phi_{2-\Delta,-J}}$$

pure gauge

	$\widetilde{A}_{\Delta,J=\pm1}$	$\widetilde{\chi}_{\Delta,J=-\frac{3}{2}}$	$\widetilde{h}_{\Delta,J=\pm2}$	
Δ	1	$\frac{3}{2}$	1	2
symmetry	large U(1)	large SUSY	supertranslation	superrotation

$\mathscr{T}_{ww}$

Virasoro

Goldstones of spontaneously broken asymptotic symmetries for $1 \leq s \leq 2$

[Donnay,AP,Strominger'18]
[Donnay,Pasterski,AP'20]
[Pasterski,AP'20]
[Pano,Pasterski,AP'21]

SUSY current

[Fotopoulos,Taylor, Stieberger,Zhu'20]

stress tensor for celestial CFT

[Kapec,Mitra,Raclariu,Strominger'16]

	$\widetilde{\psi}_{\Delta,J=-\frac{1}{2}}$	$\widetilde{A}_{\Delta,J=\pm1}$	$\widetilde{\chi}_{\Delta,J=-\frac{3}{2}}$	$\widetilde{h}_{\Delta,J=\pm2}$
Δ	$\frac{3}{2}$	2	$\frac{5}{2}$	3

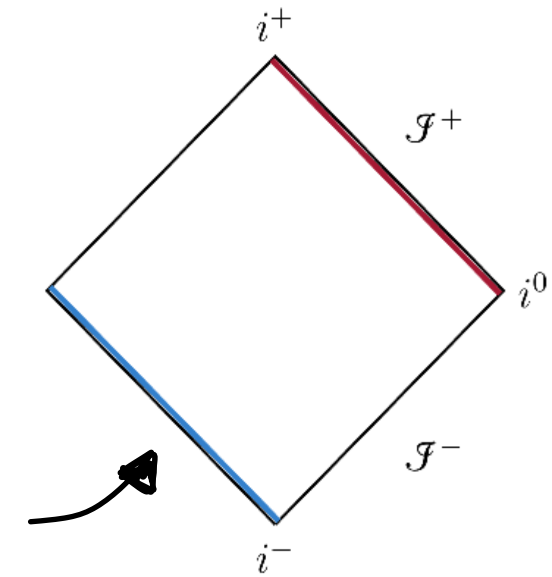
not pure gauge

Soft theorems without conformal Goldstones for $1 \leq s \leq 2$

Extrapolate-style dictionary

States of definite Δ are prepared with bulk operators $\hat{O}^s$ integrated along a light ray $\Rightarrow \mathcal{O}^s_{\Delta,J}(w,\bar{w})$ inserted at $\mathcal{I}^{\pm}$.

For fixed $(w,\bar{w})$ illustrated by antipodally placed blue (past) and red (future) contours.



Bosons:

[Pasterski,AP,Trevisani'21]

$$\mathcal{O}^{1,-}_{\Delta,+1} \propto \lim_{r\to\infty} \int du u_+^{-\Delta} \hat{A}_z(u,r,z,\bar{z}) \qquad \mathcal{O}^{2,-}_{\Delta,+2} \propto \lim_{r\to\infty} \frac{1}{r} \int du u_+^{-\Delta} \hat{h}_{zz}(u,r,z,\bar{z})$$

Fermions:

[Pano,Pasterski,AP'21]

$$\mathcal{O}^{\frac{1}{2},-}_{\Delta,+\frac{1}{2}} \propto \lim_{r\to\infty} r \int du u_+^{\frac{1}{2}-\Delta} \hat{\psi}(u,r,z,\bar{z}) \qquad \mathcal{O}^{\frac{3}{2},-}_{\Delta,+\frac{3}{2}} \propto \lim_{r\to\infty} \int du u_+^{\frac{1}{2}-\Delta} \hat{\chi}(u,r,z,\bar{z})$$

Light ray ops at future (past) null infinity create the *out* (*in*) states from vac:

$$\langle p,+| = \lim_{r\to\infty} \int du e^{i\omega(1+z\bar{z})u} \langle 0 | \hat{A}_z \quad \mapsto \quad \langle \Delta, z, \bar{z}, +| = \lim_{r\to\infty} \int du u_+^{-\Delta} \langle 0 | \hat{A}_z$$

A unified treatment of soft modes

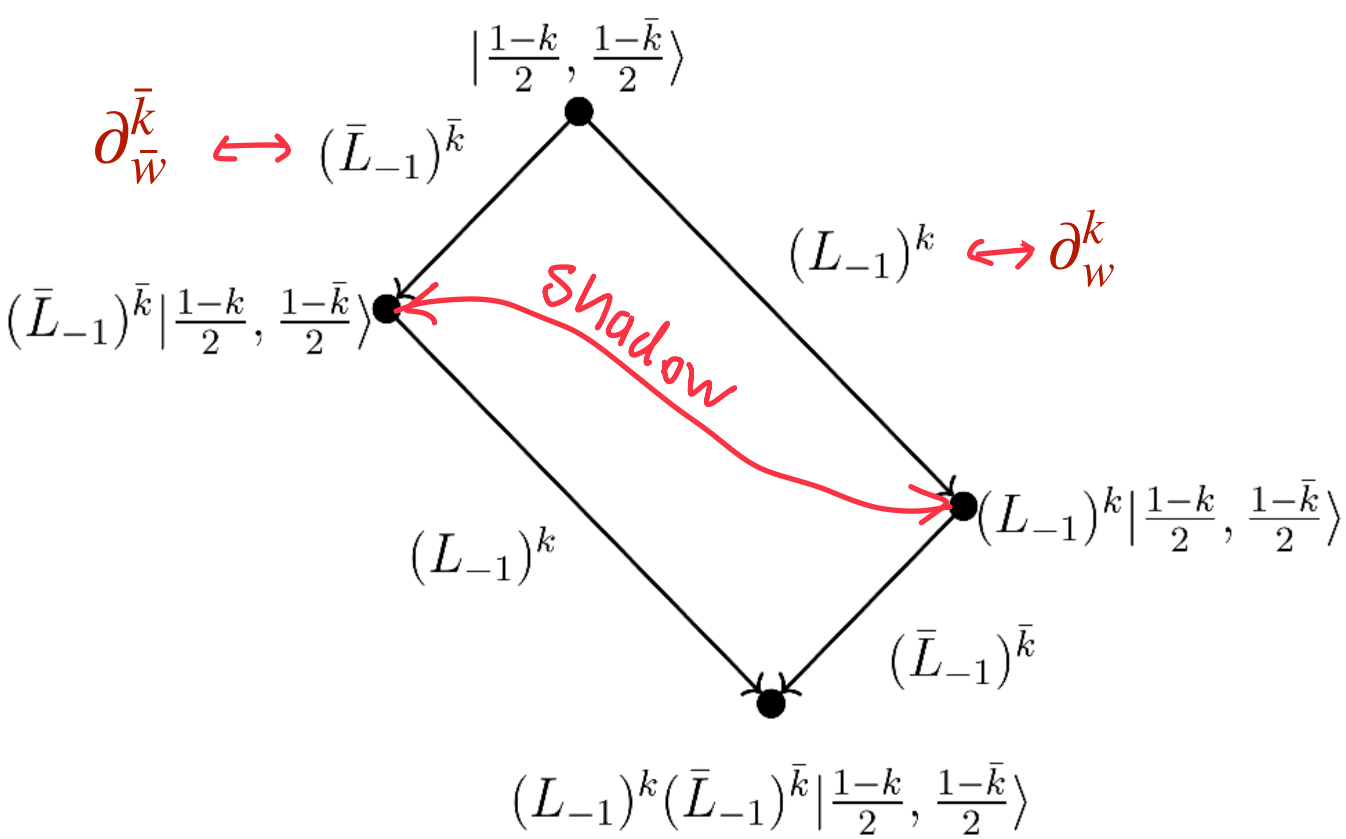
Asymptotic symmetry generators, their associated soft charges as well as conformal dressings for celestial amplitudes are all captured by the structure of the conformal multiplets of global $SL(2,\mathbb{C})$.

Primary $|h,\bar{h}\rangle$ annihilated by action of $L_1, \bar{L}_1$.

Primary descendants from action of $L_{-1}, \bar{L}_{-1}$ w/

$(L_{-1})^k|h,\bar{h}\rangle$ when $h = \frac{1-k}{2}$ with $k \in \mathbb{Z}_>$

$(\bar{L}_{-1})^{\bar{k}}|h,\bar{h}\rangle$ when $\bar{h} = \frac{1-\bar{k}}{2}$ with $\bar{k} \in \mathbb{Z}_>$



$$h = \frac{1}{2}(\Delta + J)$$

$$\bar{h} = \frac{1}{2}(\Delta - J)$$

"celestial diamonds"

[Pasterski,AP,Trevisani'21 - part 1]

Quantization: primary descendants vs null states

[Pasterski,AP,Trevisani'21 - part 1]

standard 2D CFT prescription: radial quantization

$$L_n^\dagger = L_{-n}, \quad \bar{L}_n^\dagger = \bar{L}_{-n}$$

natural prescription for celestial CFT inherited from 4D bulk

$$L_n^\dagger = -\bar{L}_n$$

IP of level-k primary descendant of $|h_2, \bar{h}_2\rangle$ with primary $\langle h_1, \bar{h}_1|$:

$$\langle h_1, \bar{h}_1 | \left(L_{-1}^k | h_2, \bar{h}_2\rangle \right) = \left(\langle h_1, \bar{h}_1 | L_1^\dagger \right) L_{-1}^{k-1} | h_2, \bar{h}_2\rangle = 0$$

$$\langle h_1, \bar{h}_1 | \left(L_{-1}^k | h_2, \bar{h}_2\rangle \right) = -\left(\langle h_1, \bar{h}_1 | \bar{L}_{-1}^\dagger \right) L_{-1}^{k-1} | h_2, \bar{h}_2\rangle \neq 0$$

→ careful not to conflate primary descendants with null states

Note: Classification of reducible modules and shadow relations independent of inner product. All we need here:

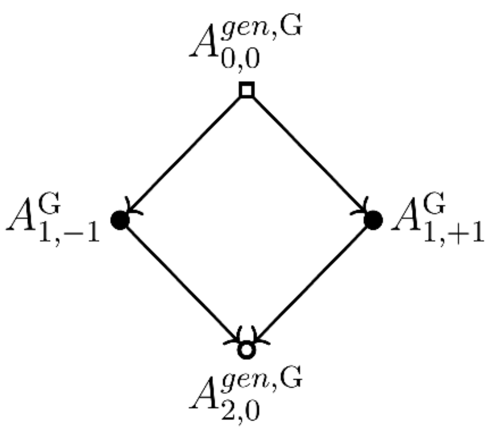
$$|h, \bar{h}\rangle \leftrightarrow \mathcal{O}_{\Delta,J} \quad \Rightarrow \quad L_{-1} \leftrightarrow \partial_w, \ \bar{L}_{-1} \leftrightarrow \partial_{\bar{w}}$$

Celestial diamonds

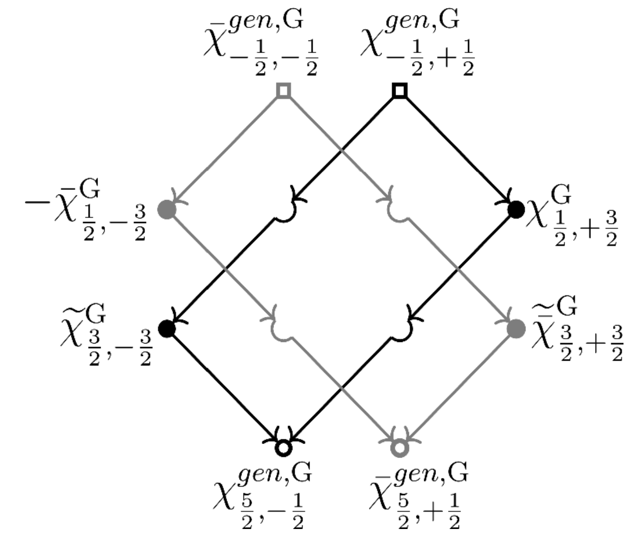
$\Phi^{gen,s} \mapsto \Phi^{s=|J|}$ or $\Phi^{s=|J|} \mapsto \Phi^{gen,s}$

[Pasterski,AP,Trevisani'21 - part 1]

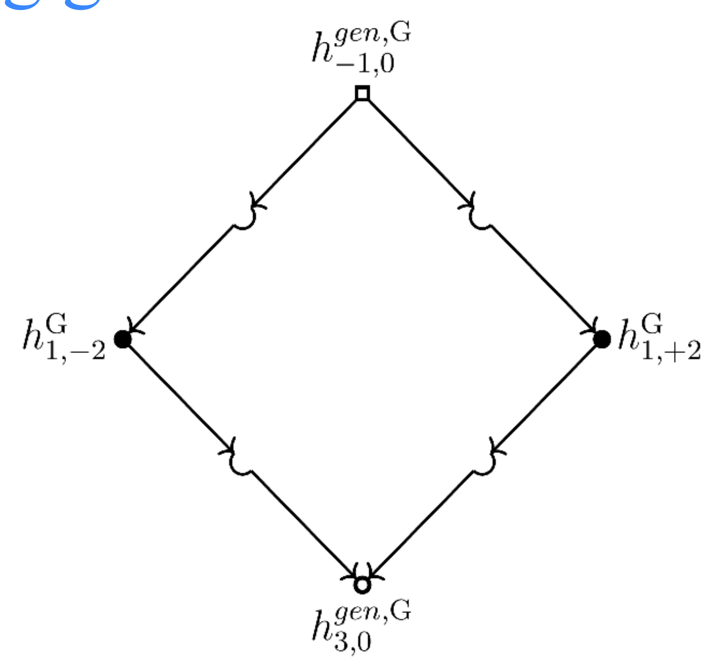
Leading photon:



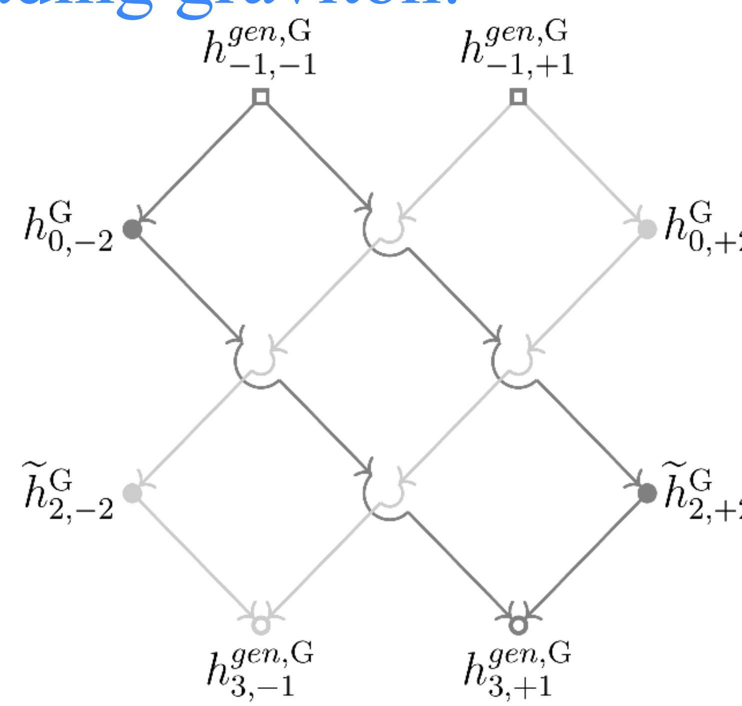
Leading gravitino:



Leading graviton:



Subleading graviton:



$\Rightarrow$ Capture most leading (conformally) soft theorems.

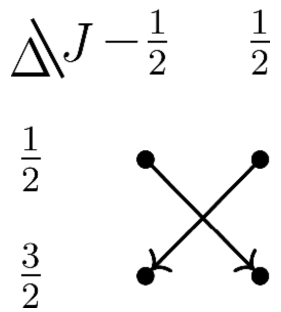
Shockwaves are at the top corner of the associated memory diamonds.

Degenerate diamonds

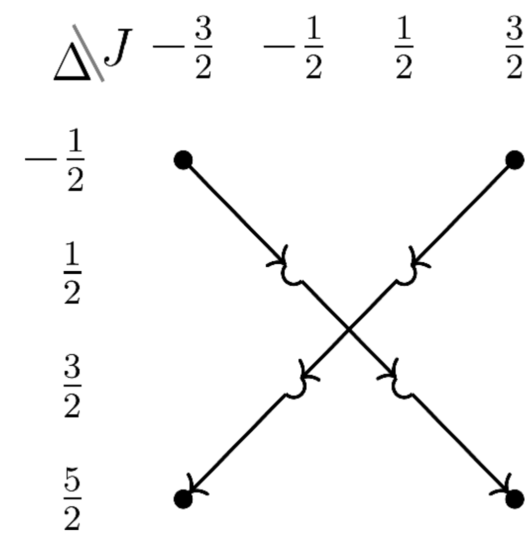
$\Phi^{s=|J|} \mapsto \widetilde{\Phi}^{s=|J|}$

[Pasterski,AP,Trevisani'21 - part 1]

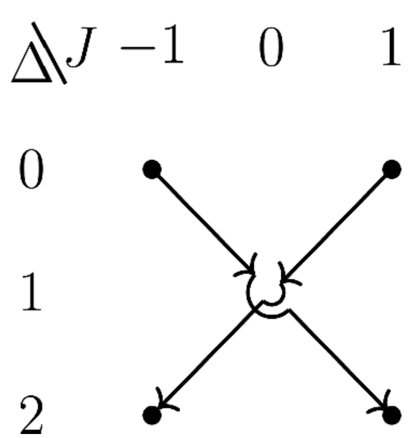
Leading photino:



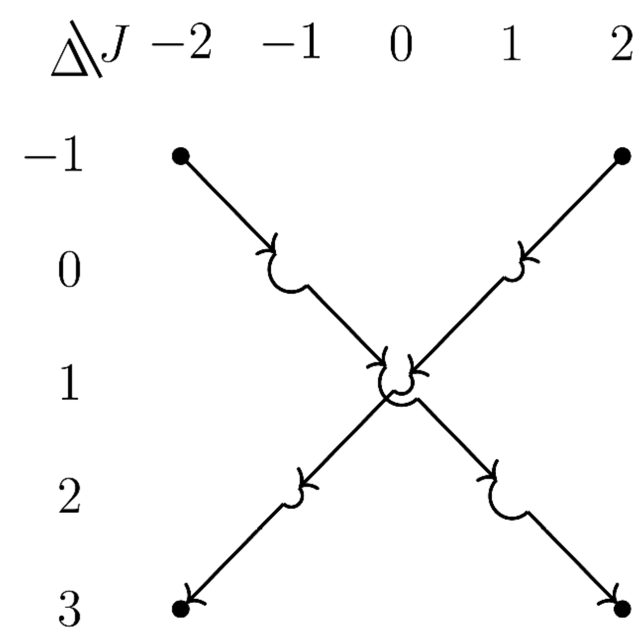
Subleading gravitino:



Subleading photon:



Subsubleading graviton:



$\Rightarrow$ Capture most subleading (conformally) soft theorems.

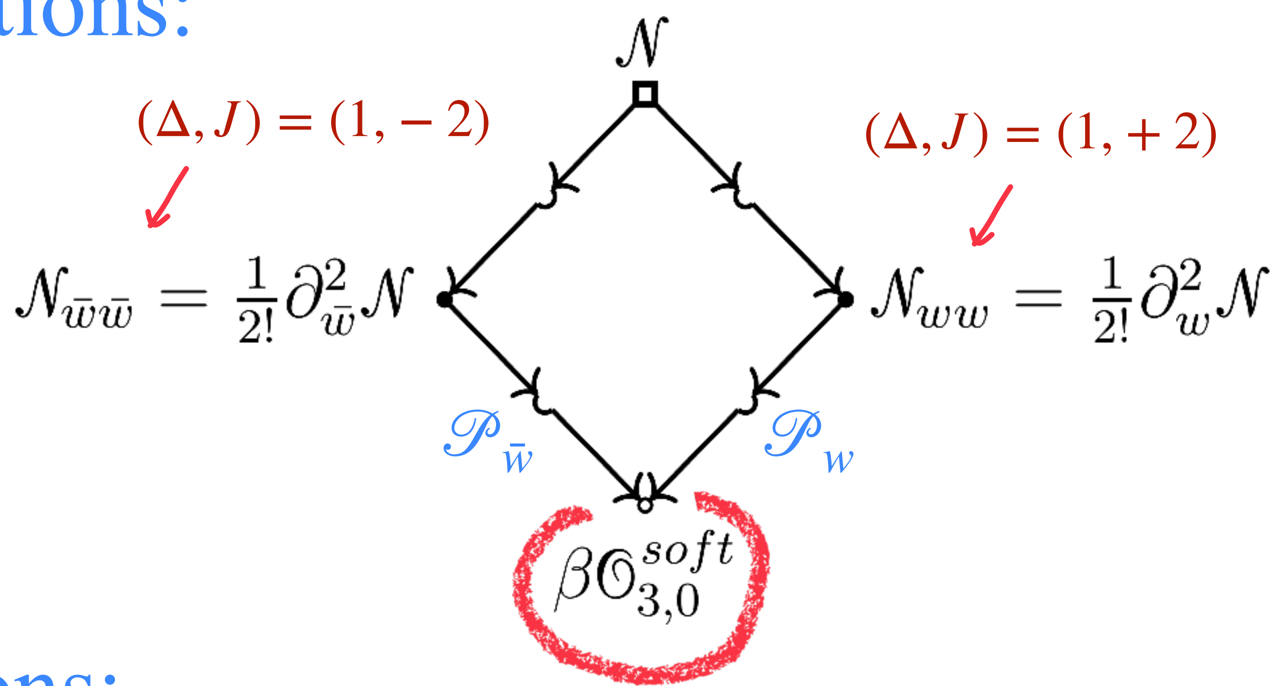
Celestial diamonds unify pure gauge & non-pure gauge soft modes.

Soft operators $\mathcal{O}^{soft}_{\Delta,J}$ in gravity

shift of bulk field

$[\mathcal{O}^{G}_{\Delta,\pm 2}, \hat{h}_{\mu\nu}] = i(\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu) = i h^{G}_{\Delta,\pm 2;\mu\nu}$

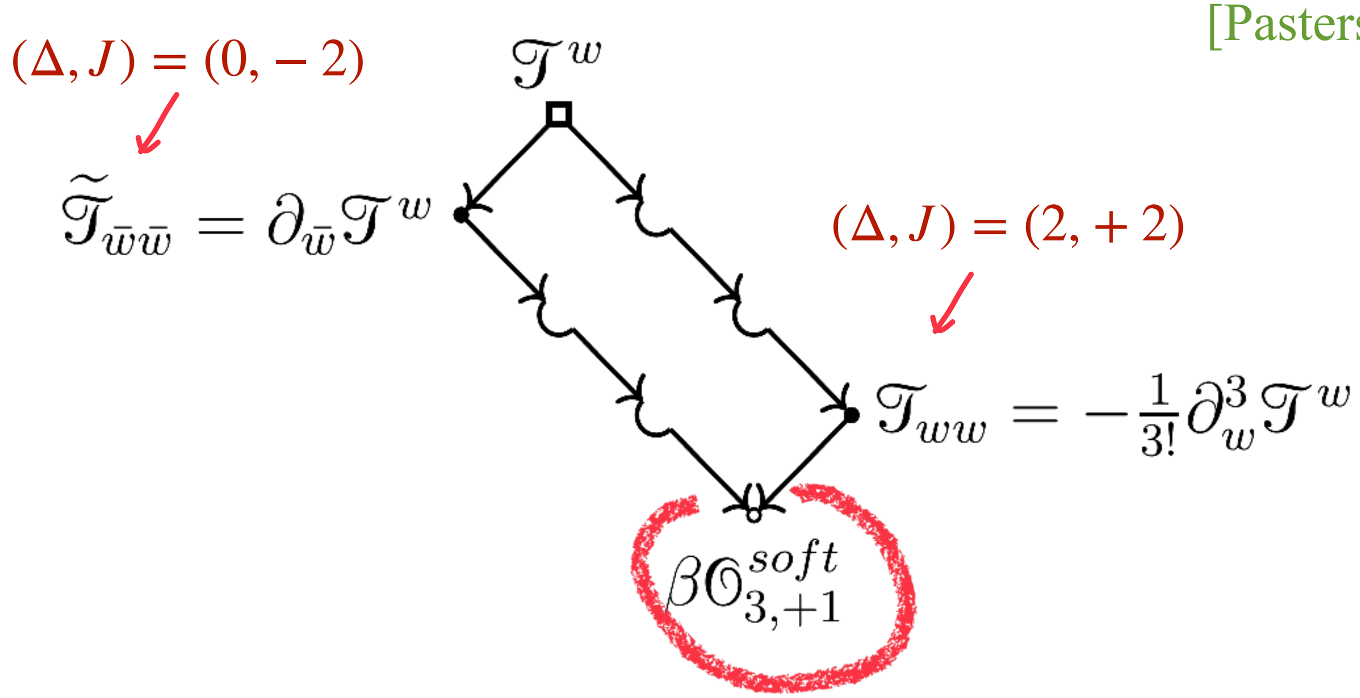
Supertanslations:



Superrotations:

[Banerjee'18]...

[Pasterski,AP,Trevisani'21 - part 2]



Soft operators $\mathcal{O}^{soft}_{\Delta,J}$ in gravity

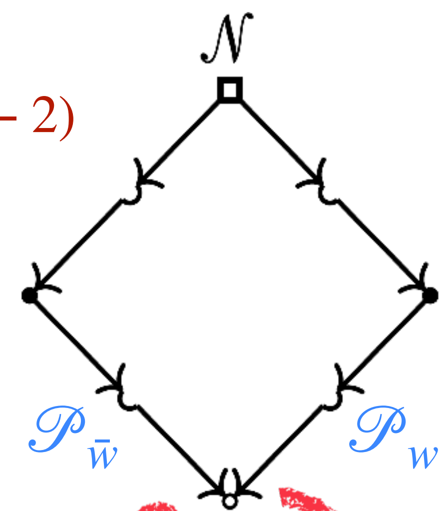
shift of bulk field

$$[\mathcal{O}^{\mathrm{G}}_{\Delta,\pm 2}, \hat{h}_{\mu\nu}] = i(\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu) = i h^{\mathrm{G}}_{\Delta,\pm 2;\mu\nu}$$

Supertanslations:

$h^{\mathrm{G}}_{1,-2;\mu\nu}$ $\quad (\Delta, J) = (1, -2) \quad\quad (\Delta, J) = (1, +2) \quad h^{\mathrm{G}}_{1,+2;\mu\nu}$

$$Q^{soft}_f = \int d^2z f_{\bar{w}\bar{w}} \mathcal{O}^{soft}_{3,0} = \mathcal{N}_{\bar{w}\bar{w}} = \frac{1}{2!}\partial^2_{\bar{w}}\mathcal{N} \qquad \mathcal{N}_{ww} = \frac{1}{2!}\partial^2_w \mathcal{N} = \int d^2z f_{ww}\mathcal{O}^{soft}_{3,0} = Q^{soft}_f$$



$$\beta \mathcal{O}^{soft}_{3,0} = \frac{\beta}{4\pi\kappa} \lim_{\Delta\to 1} (\Delta - 1)(1 + z\bar{z})\left[\partial^2_{\bar{z}}(a_{\Delta,+2} + a^\dagger_{\Delta,+2}) + \partial^2_z(a_{\Delta,-2} + a^\dagger_{\Delta,-2})\right]$$

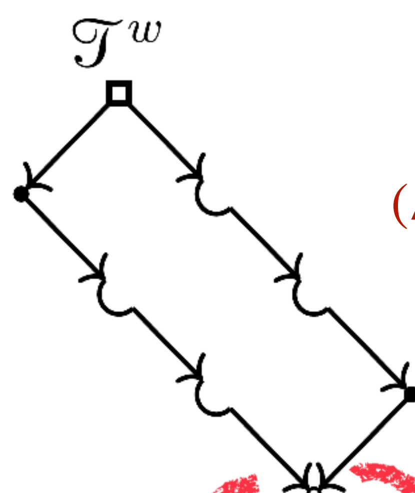
Superrotations:

[Banerjee'18]...

[Pasterski,AP,Trevisani'21 - part 2]

$(\Delta, J) = (0, -2)$

$$\widetilde{\mathcal{T}}_{\bar{w}\bar{w}} = \partial_{\bar{w}}\mathcal{T}^w$$



$(\Delta, J) = (2, +2)$

$$\mathcal{T}_{ww} = -\frac{1}{3!}\partial^3_w \mathcal{T}^w$$

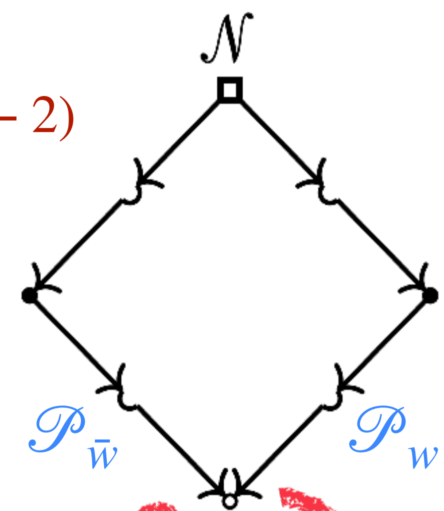
$$\beta \mathcal{O}^{soft}_{3,+1}$$

Soft operators $\mathcal{O}^{soft}_{\Delta,J}$ in gravity

shift of bulk field

$$[\mathcal{O}^{\mathrm{G}}_{\Delta,\pm2}, \hat{h}_{\mu\nu}] = i(\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu) = ih^{\mathrm{G}}_{\Delta,\pm2;\mu\nu}$$

Supertanslations:



$h^{\mathrm{G}}_{1,-2;\mu\nu}$ $\quad (\Delta, J) = (1,-2)$

$$Q^{soft}_f = \int d^2z\, f_{\bar{w}\bar{w}} \mathcal{O}^{soft}_{3,0} = \mathcal{N}_{\bar{w}\bar{w}} = \frac{1}{2!}\partial^2_{\bar{w}}\mathcal{N}$$

$(\Delta, J) = (1,+2)$ $\quad h^{\mathrm{G}}_{1,+2;\mu\nu}$

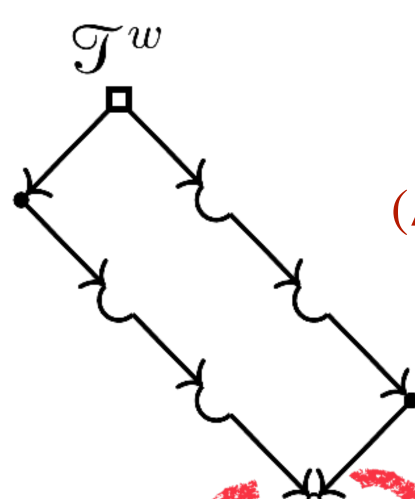
$$\mathcal{N}_{ww} = \frac{1}{2!}\partial^2_{w}\mathcal{N} = \int d^2z\, f_{ww} \mathcal{O}^{soft}_{3,0} = Q^{soft}_f$$

$$\beta\mathcal{O}^{soft}_{3,0} = \frac{\beta}{4\pi\kappa}\lim_{\Delta\to1}(\Delta-1)(1+z\bar{z})\left[\partial^2_{\bar{z}}(a_{\Delta,+2}+a^\dagger_{\Delta,+2}) + \partial^2_{z}(a_{\Delta,-2}+a^\dagger_{\Delta,-2})\right]$$

Superrotations:

[Banerjee'18]...

[Pasterski,AP,Trevisani'21 - part 2]



$h^{\mathrm{G}}_{0,-2;\mu\nu}$ $\quad (\Delta, J) = (0,-2)$

$$Q^{soft}_Y = \int d^2z\, Y^z_{\bar{w}\bar{w}} \mathcal{O}^{soft}_{3,+1} = \widetilde{\mathcal{T}}_{\bar{w}\bar{w}} = \partial_{\bar{w}}\mathcal{T}^{w}$$

Diff(S^2)

$(\Delta, J) = (2,+2)$ $\quad \widetilde{h}^{\mathrm{G}}_{2,+2;\mu\nu}$

$$\mathcal{T}_{ww} = -\frac{1}{3!}\partial^3_{w}\mathcal{T}^{w} = \int d^2z\, Y^z_{ww} \mathcal{O}^{soft}_{3,+1} = Q^{soft}_Y$$

Virasoro

$$\frac{i\beta}{4\pi\kappa}\lim_{\Delta\to0}\Delta\left[\partial^3_{z}(a_{\Delta,-2}-a^\dagger_{\Delta,-2})\right] = \beta\mathcal{O}^{soft}_{3,+1}$$

Soft operators $\mathcal{O}^{soft}_{\Delta,J}$ in gravity

shift of bulk field

$$[\mathcal{O}^{G}_{\Delta,\pm 2}, \hat{h}_{\mu\nu}] = i(\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu) = i h^{G}_{\Delta,\pm 2;\mu\nu}$$

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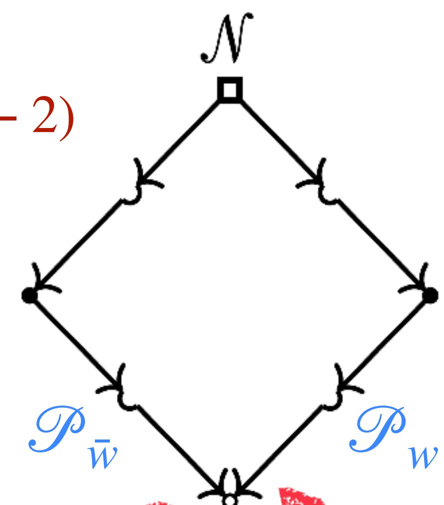

$$\mathcal{N}_{ww} = \frac{1}{2!}\partial^2_{w}\mathcal{N} = \int d^2z f_{ww} \mathcal{O}^{soft}_{3,0} = Q^{soft}_f$$

$$\beta \mathcal{O}^{soft}_{3,0} = \frac{\beta}{4\pi\kappa} \lim_{\Delta\to 1} (\Delta - 1)(1 + z\bar{z})\left[\partial^2_{\bar{z}}(a_{\Delta,+2} + a^\dagger_{\Delta,+2}) + \partial^2_{z}(a_{\Delta,-2} + a^\dagger_{\Delta,-2})\right]$$

Superrotations:

[Banerjee'18]...

[Pasterski,AP,Trevisani'21 - part 2]

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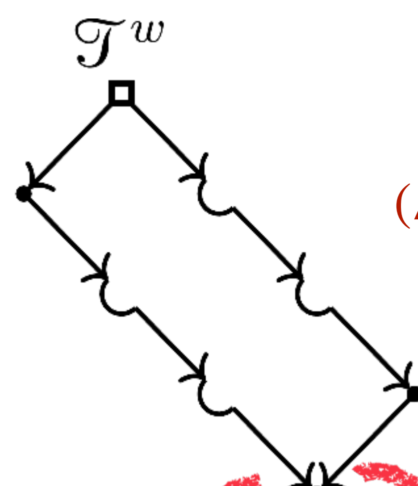
Diff(S^2)



$(\Delta, J) = (2, +2)$ $\quad \widetilde{h}^{G}_{2,+2;\mu\nu}$

$$\mathcal{T}_{ww} = -\frac{1}{3!}\partial^3_{w}\mathcal{T}^w = \int d^2z Y^z_{ww} \mathcal{O}^{soft}_{3,+1} = Q^{soft}_Y$$

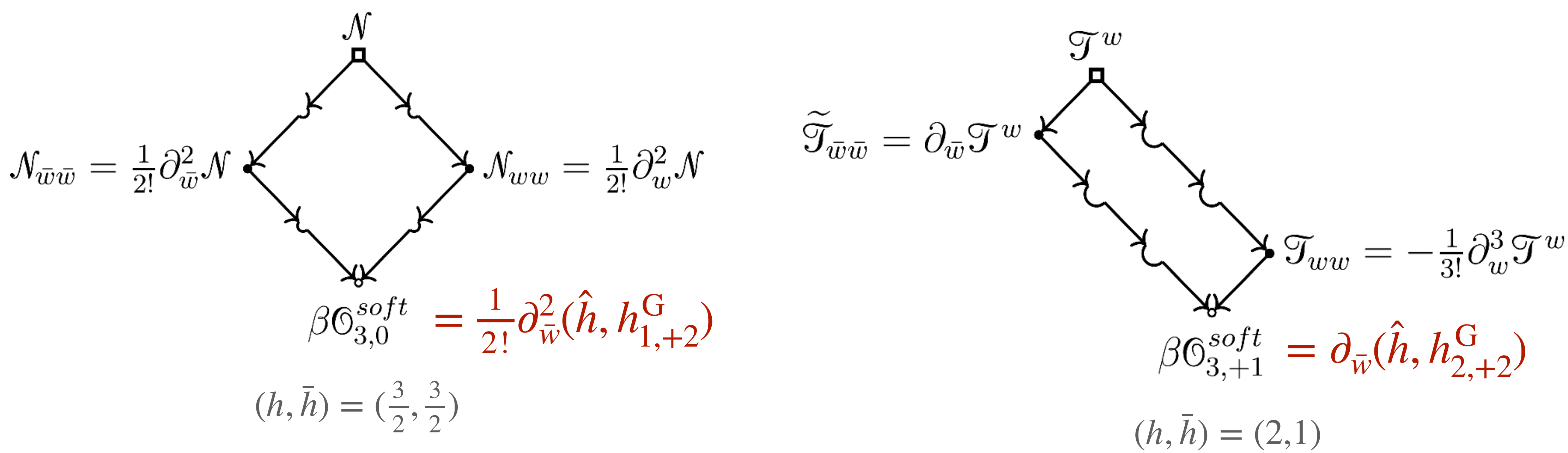
Virasoro

$$\frac{i\beta}{4\pi\kappa} \lim_{\Delta\to 0} \Delta \left[\partial^3_{z}(a_{\Delta,-2} - a^\dagger_{\Delta,-2})\right] = \beta \mathcal{O}^{soft}_{3,+1}$$

The symmetry parameters $\{f_{ww}, Y^z_{ww}, Y^z_{\bar{w}\bar{w}}\}$ are Green's fcts $\{\partial^{-2}_{\bar{z}}, \partial^{-1}_{\bar{z}}, \partial^{-3}_{z}\}$.

Memory currents

The descendants of the radiative modes appearing in $\mathcal{O}^{soft}$ are related via the constraint equation to the Bondi mass aspect m_B and the angular momentum aspect N_z $\mapsto$ associated to the bottom corners of the *memory diamonds*!



The values of Δ we find for $\mathcal{O}^{soft}$ from wavefunction approach are consistent with the super-momentum and super-angular momentum of [Barnich,Ruzziconi'21]

From representation theory it follows that they are primary descendants.

[Pasterski,AP,Trevisani'21 - part 1]

Goldstone currents

$W_{G,j}[\phi_G]\,|\,p_j\rangle$

j-th hard momentum

Fadeev-Kulish dressings render IR divergent amplitudes finite.

Conformally soft dressing (subleading order in soft expansion, leading in coupling κ): [Akhoury,Choi'19]

$$W_{G,j} = exp\left\{ \frac{\kappa}{2} \int \frac{d^3k\, \phi_G(\vec{k})}{(2\pi)^3 2k^0} \frac{p_j^\mu}{p_j \cdot k} \left[(p_j^\nu - ik_\rho J_j^{\rho\nu}) a^\dagger_{\mu\nu} - (p_j^\nu + ik_\rho J_j^{\rho\nu}) a_{\mu\nu} \right] \right\}$$

$$= exp\left\{ -i\kappa \left[\omega_j \mathscr{C} + \bar{h}_j \partial_{\bar{w}_j} \mathscr{Y}^{\bar{w}_j} + \mathscr{Y}^{\bar{w}_j} \partial_{\bar{w}_j} + h_j \partial_{w_j} \mathscr{Y}^{w_j} + \mathscr{Y}^{w_j} \partial_{w_j} \right] \right\}$$

$\phi_G(\vec{k}) = 1$

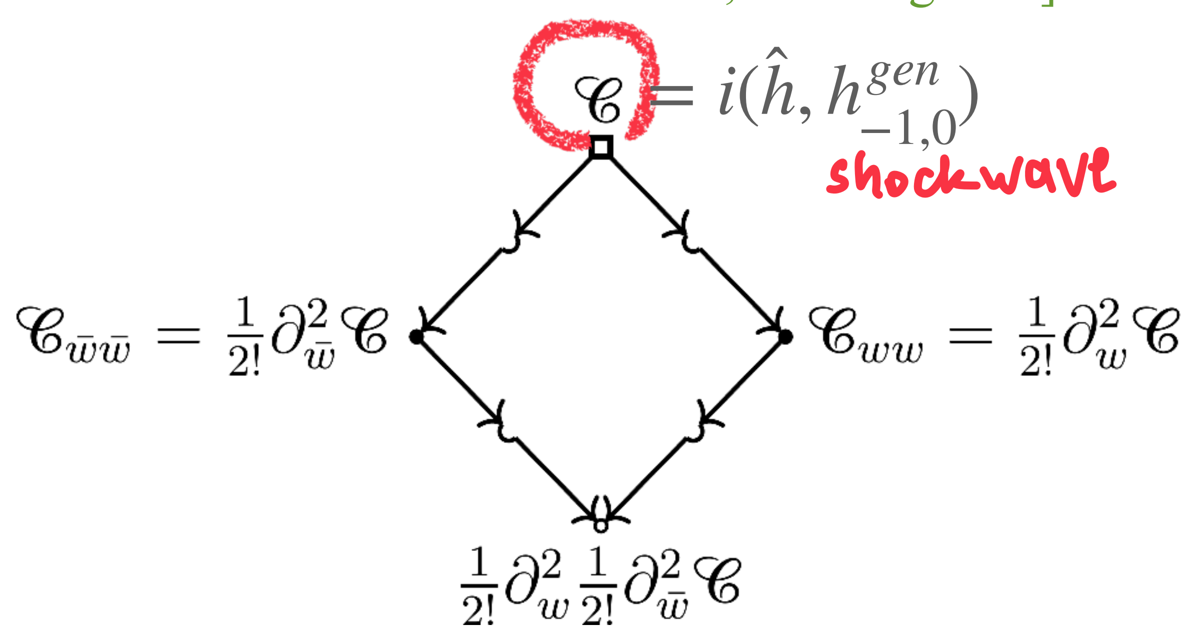
conformal dressing

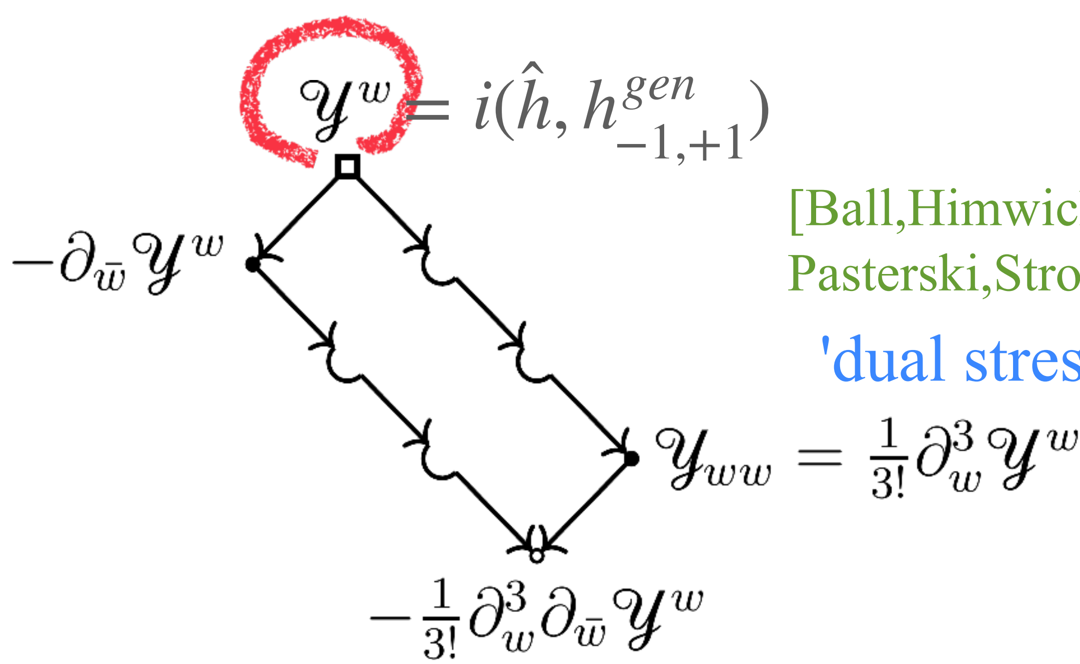
$(\Delta, J) = (-1,0)$

[Arkani-Hamed,Pate, Raclariu,Strominger'20]

$(\Delta, J) = (-1, \pm 1)$

[Pasterski,AP,Trevisani'21 - part 2]





[Ball,Himwich,Narayanan, Pasterski,Strominger'19]

'dual stress tensor'

Goldstone bosons and central extensions

The operator products between the 2D operators $\{\mathcal{N}, \mathcal{C}, \mathcal{T}^A, \mathcal{Y}^A\}$ at the top of celestial Goldstone and memory diamonds give rise to level and central extensions.

In eikonal approximation celestial amplitude factorizes into soft & hard:

$$\widetilde{\mathcal{A}} = \widetilde{\mathcal{A}}_{soft} \widetilde{\mathcal{A}}_{hard}$$

Dressing the bare hard operators with $\mathcal{C}$ cancels the IR divergent $\widetilde{\mathcal{A}}_{soft}$

$\Rightarrow$ dressed celestial amplitude $\widetilde{\mathcal{A}}_{hard}$ IR finite.

[Arkani-Hamed,Pate, Raclariu,Strominger'20]

$$\mathcal{C} = i(\hat{h}, h^{gen}_{-1,0}) \qquad \mathcal{Y}^w = i(\hat{h}, h^{gen}_{-1,+1})$$

[Pasterski,AP,Trevisani'21 - part 2]

$\downarrow$ descendancy $\downarrow$

supertranslation current $\mathcal{P}_w = i\partial^w(\hat{h}, h^{CS}_{1,-2})$ $\qquad \mathcal{Y}_{ww} = i(\hat{h}, h_{2,-2})$

'dual stress tensor'

[Ball,Himwich,Narayanan, Pasterski,Strominger'19]

$$\langle \mathcal{C}\mathcal{C} \rangle = k|w-z|^2 \log|w-z|^2 \qquad \langle \mathcal{Y}^w \mathcal{Y}^z \rangle = c(w-z)^2 \log|w-z|^2$$

$\uparrow$

[Himwich,Narayanan, Pate,Paul,Strominger'20] $\quad k = \dfrac{\kappa^2}{2\pi^2}\log\Lambda_{IR}$

chance of non-trivial central extension

2D celestial CFT toy models

[Pasterski,AP,Trevisani'21 - part 2]

Free boson $S = \int d^2w \partial_w \phi \partial_{\bar{w}} \phi$ with $\phi(w, \bar{w})$ at the top of the diamond,

not a primary but building block for constructing vertex operators $V_\alpha(w, \bar{w}) =: e^{i\alpha\phi(w,\bar{w})}$: which are charged under the symmetries generated by the primaries $\partial_w \phi$ and $\partial_{\bar{w}} \phi$ while $\partial_w \partial_{\bar{w}} \phi$ vanishes by eom.

Toy model for free limit of $\mathcal{O}^s_{\Delta,J}$ that captures features of the diamond structure is higher derivative Gaussian theory with action

$$S = \int d^2w \left[\partial^k_w \mathcal{O}^s_{\Delta,J} \partial^{\bar{k}}_{\bar{w}} \mathcal{O}^s_{\Delta,J} + \partial^k_{\bar{w}} \mathcal{O}^s_{\Delta,-J} \partial^{\bar{k}}_w \mathcal{O}^s_{\Delta,-J} \right].$$

Currents $\mathcal{J}_L \equiv \partial^{\bar{k}}_{\bar{w}} \mathcal{O}^s_{\Delta,J}$ & $\mathcal{J}_R \equiv \partial^k_w \mathcal{O}^s_{\Delta,J}$ with canonical 2pt fcts.

"Shortening condition" $\partial^k_w \partial^{\bar{k}}_{\bar{w}} \mathcal{O}^s_{\Delta,J} = 0.$

Infinite dimensional fermionic symmetries

- Soft charges for soft photino and leading soft gravitino (large SUSY)

  [Dumitrescu,He,Mitra,Strominger'14] [Avery,Schwab'15] [Lysov'15]

- Conformally soft photino/gluino theorem and leading and subleading conformally soft gravitino theorems [Fotopoulos,Stieberger,Taylor,Zhu'20]

[Pano,Pasterski,AP'21]:

2D conformally soft charge operators for photinos and gravitinos:

- extends analysis of [Donnay,AP,Strominger'18] [Donnay,Pasterski,AP'20] to infinite-dimensional fermionic symmetries
- reveals chiral structure for leading gravitino akin to subleading graviton
- offers spacetime interpretation for subleading gravitino

=> Supersymmetry stacks celestial diamonds into pyramids

From Weyl to Dirac and Majorana Primaries

Embedding photino and gravitino Weyl spinors into Dirac spinors:

$$\Psi^{s=\frac{1}{2}} = \begin{pmatrix} \psi_a \\ \bar{\psi}^{\dot{a}} \end{pmatrix}, \quad \Psi_\mu^{s=\frac{3}{2}} = \begin{pmatrix} \chi_{\mu a} \\ \bar{\chi}_\mu^{\dot{a}} \end{pmatrix},$$

+ impose Majorana condition $\bar{\psi}^{\dot{a}} = \varepsilon^{\dot{a}\dot{b}}(\psi^\dagger)_{\dot{b}}, \quad \bar{\chi}_\mu^{\dot{a}} = \varepsilon^{\dot{a}\dot{b}}(\chi^\dagger)_{\mu\dot{b}}$

Dirac spinors of definite conformal dimension and spin:

$$\Psi_{\Delta,J}^{s=\frac{1}{2}} = \begin{pmatrix} \psi_{\Delta,J} \\ \bar{\psi}_{\Delta,J} \end{pmatrix}, \quad \Psi_{\Delta,J;\mu}^{s=\frac{3}{2}} = \begin{pmatrix} \chi_{\Delta,J;\mu} \\ \bar{\chi}_{\Delta,J;\mu} \end{pmatrix},$$

Majorana spinor: $\Psi_{\mathrm{M}}(\Delta, J) = \Psi_{\Delta,J} + \Psi_{\Delta^*,-J}$

[Pano,Pasterski,AP'21]

2D fermionic operator: $\mathcal{O}^{s,\pm}_{\Delta,J}(w,\bar{w}) \equiv i(\hat{\Psi}^s(X^\mu), \Psi^s_{\Delta^*,-J}(X^\mu_\mp; w, \bar{w}))$

$$= \Omega(\hat{\Psi}^s(X^\mu), \Psi^s_{\Delta,J}(X^\mu_\pm; w, \bar{w}))$$

Soft gravitino operator

2D gravitino opertor generates shift: $\{\mathcal{O}^{s,\pm}_{\Delta,J}(w,\bar{w}),\hat{\Psi}^s(X)\} = i\Psi^s_{\Delta,J}(X_\mp;w,\bar{w})$

[Pano,Pasterski,AP'21]

$$\mathcal{O}^{s=\frac{3}{2},\pm}_{\Delta,J}(w,\bar{w}) = i\int d\Sigma_\nu(\chi_{\Delta,J;\mu}(X_\pm;w,\bar{w})\sigma^\nu\hat{\chi}^{\dagger\mu}(X)+\bar{\chi}_{\Delta,J;\mu}(X_\pm;w,\bar{w})\bar{\sigma}^\nu\hat{\chi}^\mu(X))$$

Bulk gravitino field $\hat{\chi}_{\bar{z}}$ near $\mathcal{I}^+$:

$$\lim_{r\to\infty}\hat{\chi}_{\bar{z}} = -\frac{i\kappa}{\sqrt{2}(2\pi)^2}\int_0^\infty d\omega\omega^{\frac{1}{2}}\left[a_-(\omega,z,\bar{z})e^{-i\omega(1+z\bar{z})u} - a_+^\dagger(\omega,z,\bar{z})e^{i\omega(1+z\bar{z})u}\right]|x]$$

and similarly for $\hat{\chi}^\dagger_z$ upon $a_-\mapsto a_+, a_+^\dagger \mapsto a_-^\dagger, |x]_a\mapsto |x\rangle^{\dot{a}}$ where $|x]_a = \sqrt{2}\begin{pmatrix}\bar{z}\\-1\end{pmatrix}$, $|x\rangle^{\dot{a}} = -\sqrt{2}\begin{pmatrix}1\\z\end{pmatrix}$.

($\hat{\chi}_z$ and $\hat{\chi}^\dagger_{\bar{z}}$ are subleading)

Conformally soft gravitino Weyl primaries:

$$\chi^{\mathrm{G}}_{\frac{1}{2},+\frac{3}{2};z} = \frac{-i}{(z-w)^2}\begin{pmatrix}\bar{w}\\-1\end{pmatrix}, \quad \chi^{\mathrm{G}}_{\frac{1}{2},+\frac{3}{2},\bar{z}} = 2\pi i\delta^{(2)}(z-w)\begin{pmatrix}\bar{w}\\-1\end{pmatrix},$$

$$\widetilde{\chi}^{\mathrm{G}}_{\frac{3}{2},-\frac{3}{2};z} = \pi i\delta^{(2)}(z-w)\begin{pmatrix}1\\0\end{pmatrix} - \pi i\partial_{\bar{z}}\delta^{(2)}(z-w)\begin{pmatrix}\bar{z}\\-1\end{pmatrix}, \quad \widetilde{\chi}^{\mathrm{G}}_{\frac{3}{2},-\frac{3}{2};\bar{z}} = \frac{-i}{(\bar{z}-\bar{w})^3}\begin{pmatrix}\bar{z}\\-1\end{pmatrix},$$

only one component contributes for fixed J

Soft charges for large supersymmetry

[Pano,Pasterski,AP'21]

$$\mathcal{O}_{\Delta,J}(w,\bar{w}) = i\int du d^2z \hat{\chi}_z^{\dagger(0)}\bar{\sigma}_u \chi^{G}_{\Delta,J;\bar{z}}$$

Equivalence between the large supersymmetry Ward identity and (conformally) soft gravitino theorem from $\Delta = \frac{1}{2}$ non-shadow primary:

$$\chi^{G}_{\frac{1}{2},+\frac{3}{2},\bar{z}} = 2\pi i \delta^{(2)}(z-w)\begin{pmatrix} \bar{w} \\ -1 \end{pmatrix}$$

Using $\int_0^\infty du u^{\Delta-\frac{3}{2}} = 2\pi\delta(i(\Delta-\frac{1}{2}))$ and

$$a_\pm(\omega) = \frac{1}{2\pi}\int_{1-i\infty}^{1+i\infty}(-id\Delta)\omega^{-\Delta}a_{\Delta,\pm s}$$

$$a^\dagger_\mp(\omega) = \frac{1}{2\pi}\int_{1-i\infty}^{1+i\infty}(-id\Delta)\omega^{-\Delta}a^\dagger_{\Delta,\pm s}$$

we get the soft charge: $\mathcal{O}_{\frac{1}{2},+\frac{3}{2}}(w,\bar{w}) = \frac{i\kappa}{2}\lim_{\Delta\to\frac{1}{2}}\left(\Delta-\frac{1}{2}\right)\left(a_{\Delta,+\frac{3}{2}} - a^\dagger_{\Delta,+\frac{3}{2}}\right)$

Soft charges for large supersymmetry

[Pano,Pasterski,AP'21]

$$\mathcal{O}_{\Delta,J}(w,\bar{w}) = i\int dud^2z\hat{\chi}_z^{\dagger(0)}\bar{\sigma}_u\chi^{\mathrm{G}}_{\Delta,J;\bar{z}}$$

Equivalence between the large supersymmetry Ward identity and (conformally) soft gravitino theorem from $\Delta = \frac{1}{2}$ non-shadow primary:

$$\chi^{\mathrm{G}}_{\frac{1}{2},+\frac{3}{2},\bar{z}} = 2\pi i\delta^{(2)}(z-w)\begin{pmatrix}\bar{w}\\-1\end{pmatrix}$$

Supercurrent $\mathcal{S} = \widetilde{\mathcal{O}}_{\frac{3}{2},-\frac{3}{2}}$ with $(h,\bar{h}) = (0,\frac{3}{2})$ from $\Delta = \frac{3}{2}$ shadow primary:

$$\widetilde{\chi}^{\mathrm{G}}_{\frac{3}{2},-\frac{3}{2};\bar{z}} = \frac{-i}{(\bar{z}-\bar{w})^3}\begin{pmatrix}\bar{z}\\-1\end{pmatrix}$$

$\mathcal{S} = \widetilde{\mathcal{O}}_{\frac{3}{2},-\frac{3}{2}}$ and $\widetilde{\mathcal{S}} = \mathcal{O}_{\frac{1}{2},+\frac{3}{2}}$ related by a shadow transform:

$$\widetilde{\mathcal{O}}_{\frac{3}{2},-\frac{3}{2}}(w,\bar{w}) = \frac{1}{2\pi}\int d^2w'\frac{1}{(\bar{w}-\bar{w}')^3}\mathcal{O}_{\frac{1}{2},+\frac{3}{2}}(w',\bar{w}')$$

$$\mathcal{O}_{\frac{1}{2},+\frac{3}{2}}(w,\bar{w}) = -\frac{1}{\pi}\int d^2w'\frac{(\bar{w}-\bar{w}')}{(w-w')^2}\widetilde{\mathcal{O}}_{\frac{3}{2},-\frac{3}{2}}(w',\bar{w}')$$

see also [Fotopoulos,Stieberger,Taylor,Zhu'20]

Soft operators $\mathcal{O}^{soft}_{\Delta,J}$ in supergravity

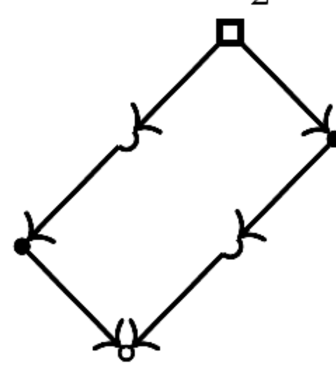
Large supersymmetry:

shift of bulk field

$$\{\mathcal{O}^{G}_{\Delta,+\frac{3}{2}}, \hat{\chi}_\mu\} = i\nabla_\mu \Lambda = i\chi^{G}_{\Delta,+\frac{3}{2};\mu}$$

[Pano,Pasterski,AP'21]

$$\mathcal{O}_{-\frac{1}{2},+\frac{1}{2}}$$



$$\mathcal{O}_{\frac{1}{2},+\frac{3}{2}} = \partial_w \mathcal{O}_{-\frac{1}{2},+\frac{1}{2}} = \tilde{\mathcal{S}}$$

$$\mathcal{S} = \widetilde{\mathcal{O}}_{\frac{3}{2},-\frac{3}{2}} = \frac{1}{2!}\partial^2_{\bar{w}}\mathcal{O}_{-\frac{1}{2},+\frac{1}{2}}$$

$$[\mathcal{O}_{soft}|x]_{\frac{5}{2},-\frac{1}{2}} = \frac{i\kappa}{8}\lim_{\Delta\to\frac{1}{2}}(\Delta-\tfrac{1}{2})(1+z\bar{z})^{-1}\partial^2_{\bar{z}}\left[a_{\Delta,+\frac{3}{2}} - a^{\dagger}_{\Delta,+\frac{3}{2}}\right]\langle x|\bar{\sigma}_u|x]$$

[Banerjee,Pandey,Paul'19]

Define fermionic analog of charge operators $\mathcal{Q}(\zeta) = \int d^2z\, \mathcal{O}_{soft}\cdot\zeta$

ζ ← ε, f, Y^z

$$\mathcal{Q}(\eta) = \int d^2z[\mathcal{O}_{soft}|\eta]$$

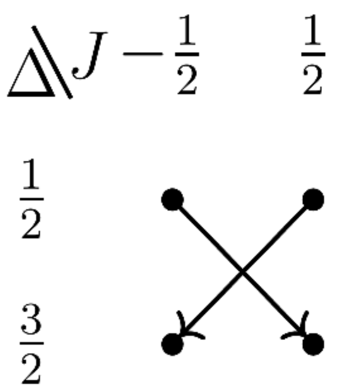
$\eta = \frac{\bar{w}-\bar{z}}{w-z}|x]$ → $\pi\mathcal{O}_{\frac{1}{2},+\frac{3}{2}} = \pi\tilde{\mathcal{S}}$

$\eta = \frac{1}{\bar{w}-\bar{z}}|x]$ → $2\pi\tilde{\mathcal{O}}_{\frac{3}{2},-\frac{3}{2}} = 2\pi\mathcal{S}$

Soft photino and subleading soft gravitino

Photino:

[Pano,Pasterski,AP'21]



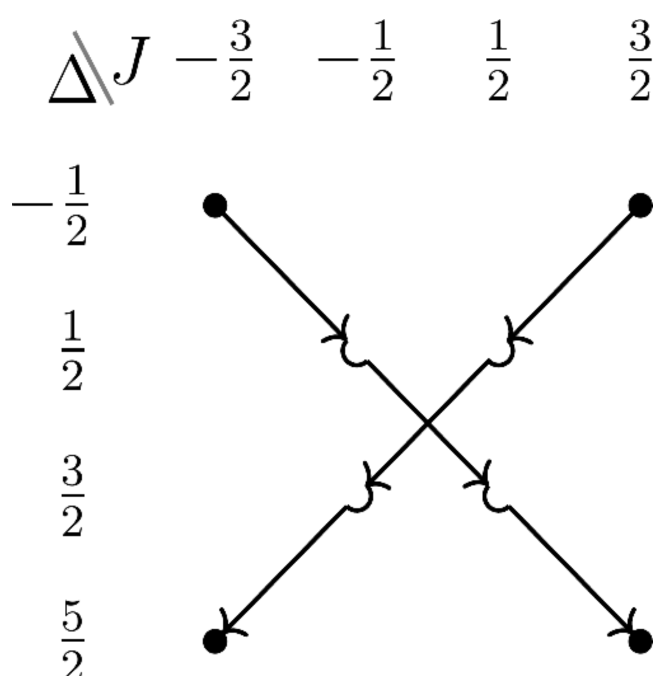
$\partial_{\bar{w}}\psi_{\frac{1}{2},+\frac{1}{2}} = -\widetilde{\psi}_{\frac{3}{2},-\frac{1}{2}} \qquad \partial_w \bar{\psi}_{\frac{1}{2},-\frac{1}{2}} = -\widetilde{\bar{\psi}}_{\frac{3}{2},+\frac{1}{2}}$

$$\mathcal{O}_{\frac{1}{2},+\frac{1}{2}} = i\int du d^2z \sqrt{\gamma}\,\hat{\psi}^{\dagger(1)}\bar{\sigma}_u \psi^{(1)}_{\frac{1}{2},+\frac{1}{2}}$$

saddle point approximation

$$\mathcal{O}^{ren}_{\frac{1}{2},+\frac{1}{2}}(w,\bar{w}) = \frac{ie}{2}\lim_{\Delta\to\frac{1}{2}}(\Delta-\tfrac{1}{2})\left(a_{\Delta,+\frac{1}{2}} - a^{\dagger}_{\Delta,+\frac{1}{2}}\right)$$

Subleading gravitino:



$\frac{1}{3!}\partial^3_{\bar{w}}\chi_{-\frac{1}{2},+\frac{3}{2}} = -\widetilde{\chi}_{\frac{5}{2},-\frac{3}{2}} \qquad \frac{1}{3!}\partial^3_{w}\bar{\chi}_{-\frac{1}{2},-\frac{3}{2}} = -\widetilde{\bar{\chi}}_{\frac{5}{2},+\frac{3}{2}}$

$$\mathcal{O}_{-\frac{1}{2},+\frac{3}{2}} = i\int du d^2z \Big\{ \sqrt{\gamma}\Big[\chi_r^{\dagger(2)}(\tfrac{1}{2}\bar{\sigma}_r - \bar{\sigma}_u)\hat{\chi}^{(0)}_{-\frac{1}{2},+\frac{3}{2};u} + \chi_u^{\dagger(1)}(\tfrac{1}{2}\bar{\sigma}_r - \bar{\sigma}_u)\hat{\chi}^{(1)}_{-\frac{1}{2},+\frac{3}{2};r}\Big]$$
$$-\hat{\chi}^{\dagger(1)}(\tfrac{1}{2}\bar{\sigma}_r - \bar{\sigma}_u)\chi^{(-1)}_{-\frac{1}{2},+\frac{3}{2};z} + \underbrace{\hat{\chi}_z^{\dagger(0)}\bar{\sigma}_u\chi^{(0)}_{-\frac{1}{2},+\frac{3}{2};\bar{z}}}\Big\}$$

saddle point approximation

$$\mathcal{O}^{ren}_{-\frac{1}{2},+\frac{3}{2}}(w,\bar{w}) = \frac{\kappa}{2}\lim_{\Delta\to\frac{1}{2}}(\Delta+\tfrac{1}{2})\left(a_{\Delta,+\frac{3}{2}} + a^{\dagger}_{\Delta,+\frac{3}{2}}\right)$$

Celestial pyramids s-direction

[Pano,Pasterski,AP'21]

$\mathcal{N}=1$ supercharges in Mellin basis:

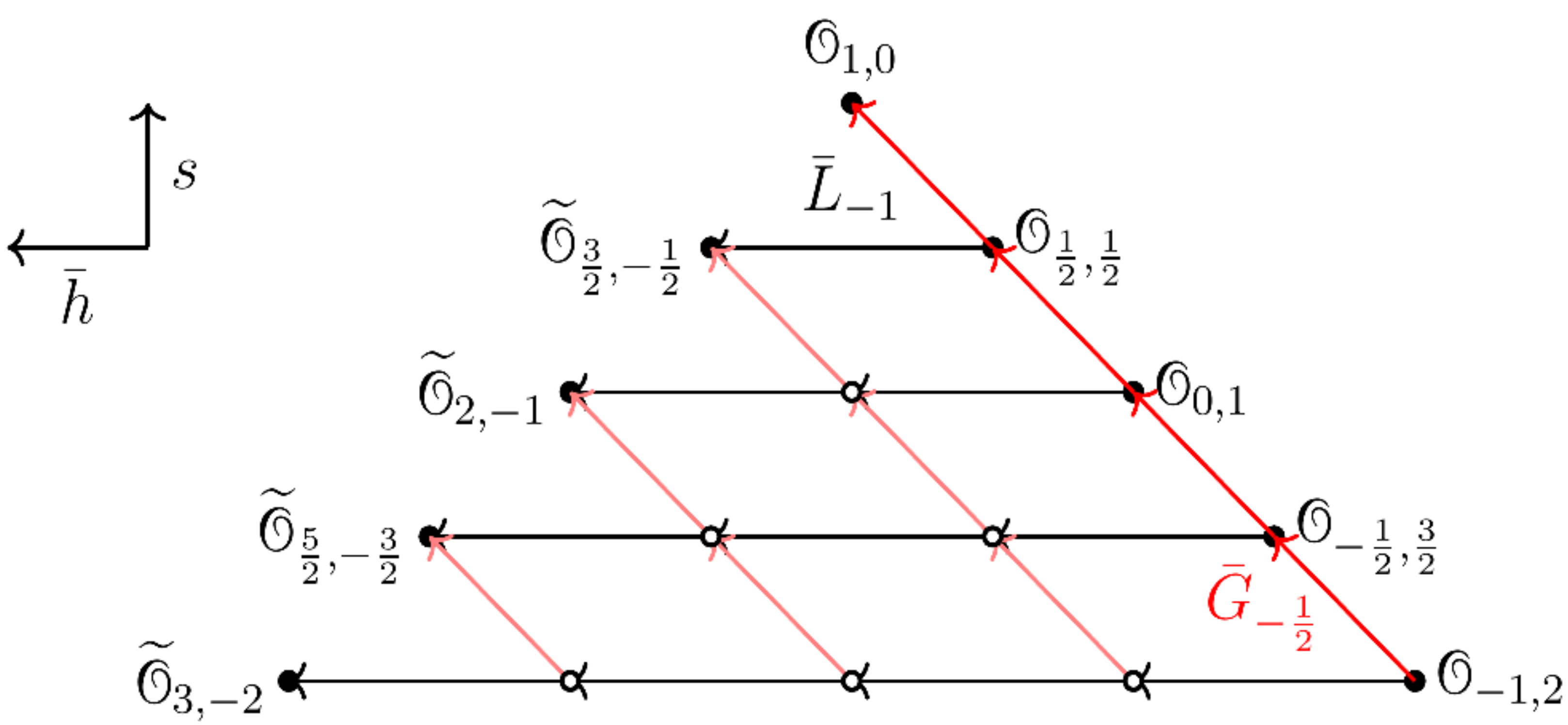
$$Q=\partial_\theta |q\rangle e^{\partial_\Delta/2} \qquad \bar{Q}=\theta|q]e^{\partial_\Delta/2} \qquad \{\bar{Q},Q\}=\sigma^\mu \mathcal{P}_\mu$$

How do supercharges interact with SL(2,ℂ) multiplet structure?

$$[\partial_{\bar{w}},Q]=[\partial_w,\bar{Q}]=0 \quad \Leftrightarrow \quad [\bar{L}_m,G_n]=[L_m,\bar{G}_n]=0$$

$$\bar{G}_{-\frac{1}{2}}\mathcal{O}^s_{\Delta,s}=\mathcal{O}^{s-\frac{1}{2}}_{\Delta+\frac{1}{2},s-\frac{1}{2}}$$

[Fotopoulos,Stieberger,Taylor,Zhu'20]



vertical section of the celestial pyramid

scalar
photino
photon
gravitino
graviton

Celestial pyramids (Δ, J) plane

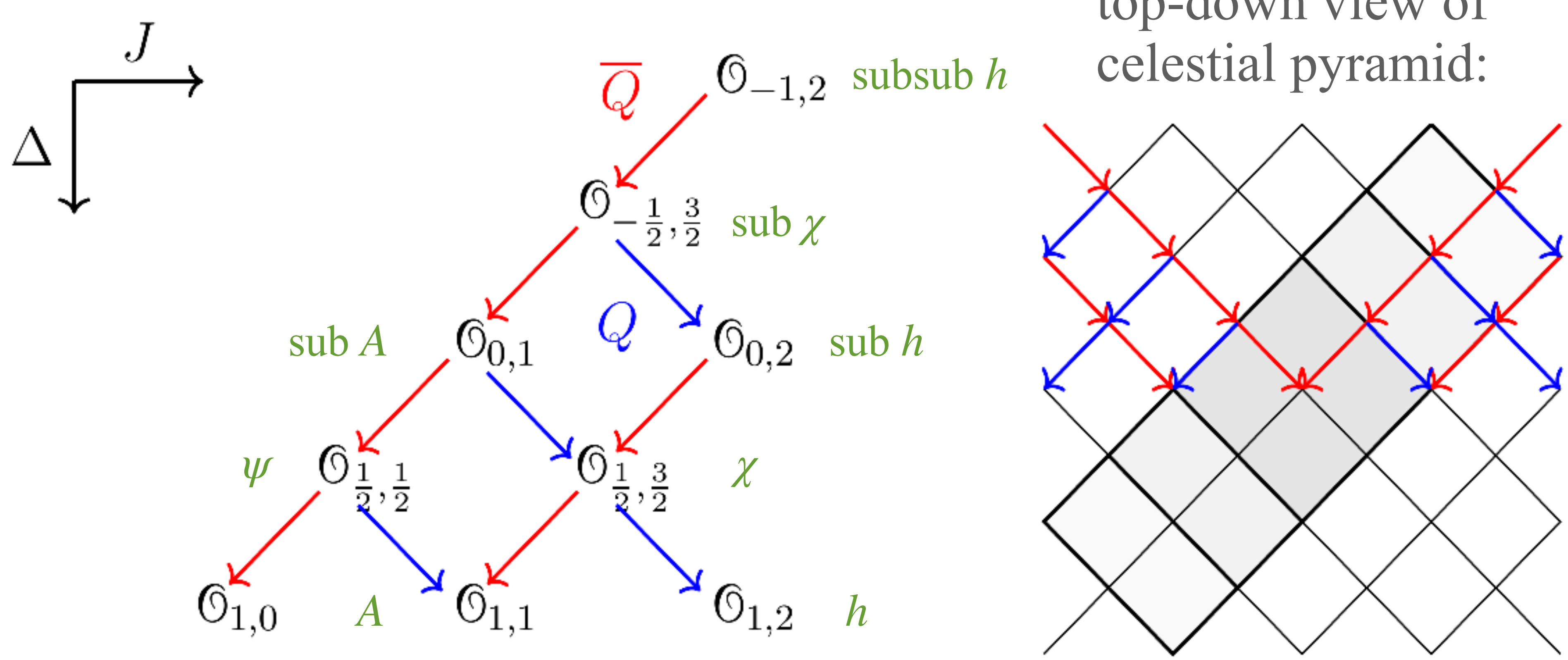
[Pano,Pasterski,AP'21]

Spin shifting relation $\bar{G}_{-\frac{1}{2}}\mathcal{O}^s_{\Delta,s} = \mathcal{O}^{s-\frac{1}{2}}_{\Delta+\frac{1}{2},s-\frac{1}{2}}$ implies for soft charges:

$$\bar{k}\bar{G}_{-\frac{1}{2}}\mathcal{O}^s_{soft} = \bar{L}_{-1}\mathcal{O}^{s-\frac{1}{2}}_{soft}$$

w/ $\mathcal{O}^s_{soft} = \frac{1}{\bar{k}!}\partial^{\bar{k}}_{\bar{w}}\mathcal{O}^s_{\Delta,J}$ & $\bar{k} = 1 + J - \Delta$

top-down view of celestial pyramid:



The SUSY generators shift between radiative primaries corresponding to conformally soft theorems within the same helicity sector ($\mathcal{N} > 1$ SUSY for full pyramid).

Celestial currents in the pyramid

[Pano,Pasterski,AP'21]

The global SL(2,ℂ) descendants shift within the (Δ, J) plane while the SUSY charges translate in the transverse s-direction.

Spin shifting relation for shadow operators:

$$(1+J-\Delta)\bar{G}_{-\frac{1}{2}}\underbrace{\tilde{\mathcal{O}}^{s}_{2-\Delta,-s}}_{\mathcal{T}} = -\bar{L}_{-1}\underbrace{\tilde{\mathcal{O}}^{s-\frac{1}{2}}_{\frac{3}{2}-\Delta,\frac{1}{2}-s}}_{\mathcal{S}}$$

e.g. $\Delta=0, s=2$

Celestial $s = 1,\frac{3}{2},2$ symmetry currents:

$$\tilde{\mathcal{O}}_{1,-1},\ \tilde{\mathcal{O}}_{\frac{3}{2},-\frac{3}{2}},\ \tilde{\mathcal{O}}_{2,-2}$$

$$\mathcal{J} \qquad \mathcal{S} \qquad \mathcal{T}$$

large U(1), large SUSY, Virasoro superrotation

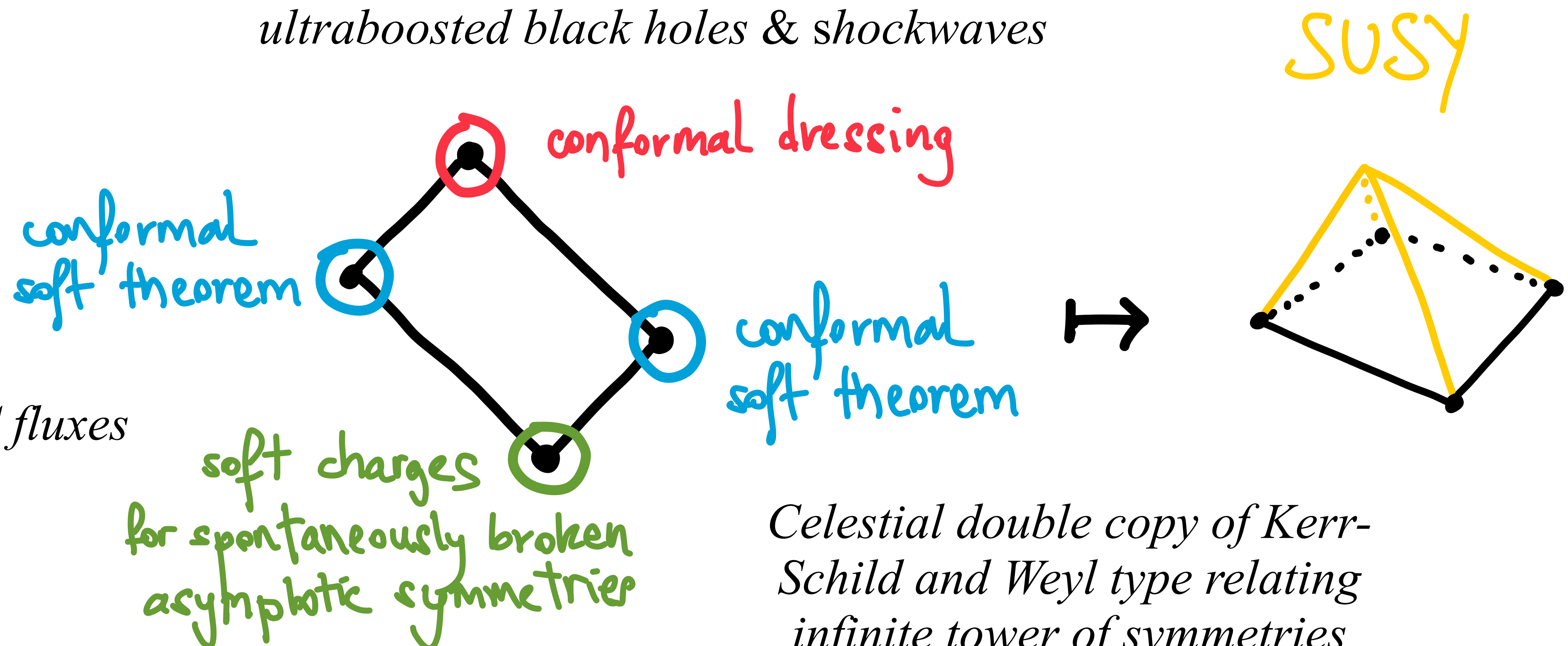
The symmetry parameters ε η Y^z are all $\propto \dfrac{1}{\bar{z}-\bar{w}}$

since their first ∂_w descendants are the respective soft charges.

Summary

Celestial diamonds reveal power of symmetry to organize conformally soft physics. Stack via SUSY into **celestial pyramids**.

Exact sol to Einstein eq:
ultraboosted black holes & shockwaves



BMS fluxes

Celestial double copy of Kerr-Schild and Weyl type relating infinite tower of symmetries

↦ ***Combining tools from amplitudes, classical GR & CFT:***

insight into bulk physics from Celestial Holography programme.

Exploration of celestial territory has only begun!

Thank you!